



# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

## B.Sc. DEGREE EXAMINATION – PHYSICS

THIRD SEMESTER – APRIL 2018

### MT 3100- ALLIED MATHEMATICS FOR PHYSICS

Date: 05-05-2018  
Time: 09:00-12:00

Dept. No.

Max. : 100 Marks

#### PART A

Answer ALL the questions

(10 x 2 = 20)

1. If  $y = (ax + b)^m$ , find  $y_n$ .
2. Find the polar subtangent and subnormal of the curve  $r = a\theta$
3. Write the expansion for  $\log\left(\frac{1+x}{1-x}\right)$ .
4. Find the rank of the matrix  $A = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{pmatrix}$ .
5. Find the Laplace transform of  $e^{2t} + 3e^{-5t}$ .
6. Find  $L^{-1}\left(\frac{1}{(s+1)^2}\right)$ .
7. Write down the expansion of  $\tan\theta$  in a series of ascending powers of  $\theta$ .
8. Prove that  $\cosh^2x - \sinh^2x = 1$ .
9. What is the chance that a leap year selected at random will contain 53 Sundays?
10. Write down the probability mass function for the Poisson distribution.

#### PART B

Answer any FIVE questions

(5 x 8 = 40)

11. Find the  $n^{\text{th}}$  differential coefficient of  $x^2e^{3x}$ .
12. Find the lengths of the subtangent and subnormal at  $(a, a)$  on the cissoid  $y^2 = \frac{x^3}{2a-x}$ .
13. If  $a, b, c$  denote three consecutive integers, show that  $\log_e b = \frac{1}{2}\log_e a + \frac{1}{2}\log_e c + \frac{1}{2ac+1} + \frac{1}{3(2ac+1)^3} + \dots$
14. Verify Cayley Hamilton theorem for the matrix  $A = \begin{pmatrix} 2 & -1 \\ -8 & 4 \end{pmatrix}$ .
15. Find the Laplace transform of (i)  $\frac{e^{-3t}-e^{-4t}}{t}$  (ii)  $t \cos 3t$ .
16. Write down the expansion of  $\cos 5\theta$  in terms of  $\cos\theta$ .
17. Calculate the mean for the following table giving the age distribution of 542 members:

Age(in years)	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of members	3	61	132	153	140	51	2

18. Ten coins are thrown simultaneously. Find the probability of getting at least seven heads.

#### PART C

Answer any TWO questions

(2 x 20 = 40)

19. (a) Find the angle of intersection of the cardioids  $r = a(1 + \cos\theta)$  and  $r = b(1 - \cos\theta)$ .

(b) Find the maximum and minimum values of the function  $f(x) = 2x^3 - 3x^2 - 36x + 10$ .  
(10+10)

20. (a) Find the eigenvalues and eigenvectors of the matrix  $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ .

(b) Find the sum to infinity of the series  $1 + \frac{3}{4} + \frac{3 \cdot 5}{4 \cdot 8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + \dots$ . (12+8)

21. (a) Solve the equation  $y'' - 3y' + 2y = e^{2t}$ , given that  $y(0) = -3$ ,  $y'(0) = 5$ .

(b) Prove that  $\cos^6 \theta = \frac{1}{32} [\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10]$ . (12+8)

22. (a) Separate into real and imaginary parts of  $\tan(x + iy)$ .

(b) Two unbiased dice are thrown. Find the probability that (i) both the dice show the same number (ii) the first die shows 6 (iii) the total of the numbers on the dice is 8 (iv) the total of the numbers on the dice is greater than 8 (v) the total of the numbers on the dice is 13.

(8+12)

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