



Date: 05-05-2018

Dept. No. 

Max. : 100 Marks

Time: 09:00-12:00

**PART-A****Answer ALL the questions:**

(10 x 2=20)

1. If  $y = \frac{x+1}{x+2}$  find  $\frac{d^2y}{dx^2}$ .

2. Define an orthogonal matrix.

3. Show that the matrix  $A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$  is unitary.

4. Write down the middle term of  $\left(x - \frac{2}{x}\right)^{12}$ .

5. Find the value of  $L[\sinh 17t]$ .

6. Prove that  $\frac{a-x}{a} + \frac{1}{2} \left(\frac{a-x}{a}\right)^2 + \frac{1}{3} \left(\frac{a-x}{a}\right)^3 + \dots = \log a - \log x$ .

7. Find the value of  $L^{-1} \left[ \frac{1}{s-3} + \frac{1}{s} \right]$ .

8. Write down the expansion for  $\sin n\theta$ .9. If  $\sin^2\theta + \cos^2\theta = 1$ , Show that  $\cosh^2 x - \sinh^2 x = 1$ .10. If a Poisson variate  $X$  is such that  $P(X = 1) = 2P(X = 2)$ . Find the mean.**PART-B****Answer any FIVE questions:**

(5 x 8=40)

11. Find the  $n^{\text{th}}$  differential coefficient of  $\cos x \cos 2x \cos 3x$ .12. Find the sum to infinity of the series  $1 + \left(\frac{3}{1.2}\right) + \left(\frac{5}{1.2.3}\right) + \left(\frac{7}{1.2.3.4}\right) + \dots \infty$ .13. Find the Eigen values and Eigen vectors of the matrix  $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ .14. If  $\sin(A + iB) = x + iy$ , thenProve that (i)  $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$  (ii)  $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$ .

15. Find  $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$ .

16. Find (i)  $L[\cosh t \cdot \sin 2t]$ , (ii)  $L[t \cdot e^{-t} \cdot \sin t]$ .

17. Find the extreme values of the function  $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ .

18. From a well-shuffled pack of 52 cards, one card is drawn at random. What is the probability that it will be (i) a jack (ii) a spade (iii) a club (iv) a heart?

### PART - C

**Answer any TWO questions:**

**(2 x 20=40)**

19. Verify Cayley – Hamilton theorem for the matrix  $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ .

20. (a) If  $y = \sin(m \sin^{-1} x)$  then prove that  $(1 - x^2)y_2 - xy_1 + m^2y = 0$  and hence

prove that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 + n^2)y_n = 0$ .

(b) Find the mean and standard deviation for the following table, giving the age distribution of 542 members.

Age in years	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of members	3	61	132	153	140	51	2

(10+10)

21. (a) Solve the equation  $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 5y = 5$  given that  $y = 0, \frac{dy}{dt} = 2$  when  $t = 0$ .

(b) Find  $L\left[\frac{1-e^t}{t}\right]$ . (15 + 5)

22. (a) Prove that  $\frac{\sin 7\theta}{\sin \theta} = 64\cos^6\theta - 80\cos^4\theta + 24\cos^2\theta - 1$ .

(b) Find the angle at which the radius vector cuts the curve  $\frac{l}{r} = 1 + e \cos\theta$ . (10+10)

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