LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2018**

MT 3503– VECTOR ANALYSIS & ORDINARY DIFF. EQUATIONS

Date: 05-05-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

Answer ALL questions: **(10 x 2 = 20 marks)**

1. Find the unit vector normal to the surface $x^2 + y^2 + 2z^2 = 4$ at the point (1,1,1).
2. If $ax^2z^2\bar{i} + xyz^2\bar{j} - xy^3\bar{k}$ is solenodal, find the value of a.
3. If $\bar{F} = x\bar{j} - y\bar{i}$, Compute $\int_c \bar{F}.d\bar{r}$ along the straight line joining (0,0) and (1,1).
4. Define conservative field and scalar potential.
5. State Stoke's theorem.
6. For any closed surface S, prove that $\iint_s curl\,\bar{F}.\hat{n}\,ds = 0$.
7. Solve: $\frac{1}{x}\frac{dy}{dx} + \frac{y}{x}\tan x = \cos x$.
8. Solve: $(y - px)(p - 1) = p$.
9. Solve: $(D^3 - 3D^2 + 4)y = 0$.
10. Transform the equation $(2x+3)^2\, y'' - 2(2x+3)y' + 2y = 6x$ into a linear equation with constant coefficients.

PART – B

Answer any FIVE questions: **(5 x 8 = 40 marks)**

11. Determine *f*(r) so that the vector $f(r)\bar{r}$ is solenoidal.
12. Find ϕ *given that* $\phi(1,1,1) = 3$ and $\nabla\phi = (y + y^2 + z^2)\bar{i} + (x + y + 2xy)\bar{j} + (y + 2zx)\bar{k}$.
13. If $\bar{F} = (2x^2 - 3z)\bar{i} - 2xy\bar{j} - 4z\bar{k}$, then evaluate $\iiint_V \nabla.\bar{F}dv$, where V is bounded by the planes $x = 0, y = 0, z = 0$, and $2x + 2y + z = 4$.

14. Using Green's theorem, evaluate $\int_c (y-\sin x)dx+\cos x dy$, where C is the triangle formed by y=0,

$x=\frac{\pi}{2}, y=\frac{2}{\pi}x.$

15. Solve: $(1-x^2)\frac{dy}{dx}+xy=y^3\sin x.$

16. Solve: $p(p+y)=x(x+y).$

17. Solve: $(D^2+2D-3)y=e^x\cos x+e^{-2x}$.

18. Using variation of parameters, solve $y''+a^2y=\sec(ax).$

PART – C

Answer any TWO questions: **(2 x 20 = 40 marks)**

19. a) If $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$, prove that div $(gradr^n)= n(n+1)r^{n-2}$.

b) If $\bar{F}=2y\bar{i}-z\bar{j}+x\bar{k}$, evaluate $\vec{F}x\vec{dr}$ along the curve

$x=\cos t, y=\sin t, z=2\cos t \ \ from t=0 \ to \ t=\frac{\pi}{2}.$ **(10+10)**

20. Verify stoke's theorem for $\bar{F}=(2x-y)\bar{i}-yz^2\bar{j}-y^2z\bar{k}$, where S is the upper half of the sphere

$x^2+y^2+z^2=1$ and C is its boundary. **(20)**

21. a) Solve : $\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sec y.$

b) Solve : $y=2px+y^2p^3.$ **(10 +10)**

22. a) Solve: $(D^2-2D+1)y=xe^x\sin x.$

b) Solve: $(3x+2)^2y''+3(3x+2)y'-36y=3x^2+4x+1.$ **(10 +10)**

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