LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRDSEMESTER – APRIL 2018

MT 3810- TOPOLOGY

Date: 30-04-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all the questions. Each question carries 20 marks.

I. a)1) In any metric space prove that each closed sphere is a closed set.

OR

a)2) Define the following: (i)topological space (ii) metrizable space and (iii) relative topology. (5)

b)1) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that
f is continuous $\Leftrightarrow$ $f^{-1}(G)$ is open whenever G is open in Y.

b)2) State and prove Cantor's intersection theorem. (7+8)

OR

c)1) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous at x_o if and only if $x_n \to x_o \Rightarrow f(x_n) \to f(x_o)$

c)2) If $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X then prove that there exists a point in X which is not in any of the A_n's. (7+8)

II. a)1) Prove that any continuous image of a compact space is compact.

OR

a)2) Prove that any closed subspace of a compact space is compact. (5)

b)1) State and prove Lindelof's theorem.

b)2) Prove that a topological space is compacat iff every class of closed sets with the finite intersection property has non-empty intersection. (7+8)

OR

c) Prove that a topological space is compact if every class of subbasic closed sets with the finite intersection property has non-empty intersection. (15)

III.a)1) Prove that a one-to-one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism.

OR

a)2) Quoting the necessary results prove that a metric space is compact implies it is complete and totally bounded. (5)

b)1) Prove that the product of any non-empty class of Hausdorff space is a Hausdorff space.

b)2) State and prove Ascoli's theorem. (3+12)

OR

c) Prove that the following statements are equivalent:
(i) X is compact (ii) X is sequentially compact and (iii) X has the Bolzano-Weierstrass property. (15)

IV.a)1) Prove that any continuous image of a connected space is connected.

OR

a)2) Prove that the product of any non-empty class of connected space is connected. (5)

b) State and prove Tietze Extension theorem.

OR

c) State and prove Urysohn Imbedding theorem. (15)

V.a)1) State Real and Complex Stone Weierstrass theorems.

OR

a)2) Prove that X_∞ is compact. (5)

b) Proving the necessary lemmas, state and prove Real Stone-Weierstrass theorem.

OR

c) State and prove Weierstrass approximation theorem. (15)

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