LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **PHYSICS**

FOURTHSEMESTER – APRIL 2018

MT 4200- ADVANCED MATHEMATICS FOR PHYSICS

Date: 02-05-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

SECTION A

Answer **ALL** the questions: **(10x2 = 20)**

1. Evaluate $\int\left(x+\frac{1}{x}\right)^2 dx$.
2. Write any two properties of definite integrals.
3. Define exact differential equation.
4. Solve $\frac{dy}{dx}=\frac{y+2}{x-1}$.
5. Evaluate $\int_0^a\int_0^b\left(x^2+y^2\right)dxdy$.
6. Prove that $\beta(m,n)=\beta(n,m)$.
7. Prove that $\nabla.r=3$ and $\nabla\times r=0$.
8. State Gauss Divergence theorem.
9. Define a cyclic group and give an example.
10. Define Kronecker's delta.

SECTION B

Answer any **FIVE** questions: **(5x8 = 40)**

11. Evaluate $\int(\log x)^2\,dx$ using integration by parts method.
12. Prove that $\int_0^{\pi}\theta\sin^3\theta\,d\theta=\frac{2\pi}{3}$.
13. Solve $y^2+x^2\frac{dy}{dx}=xy\frac{dy}{dx}$.
14. Solve $(D-1)^2 y=x$.
15. Evaluate $\iint\left(x^2+y^2\right)dxdy$ over the region for which x, y are each ≥ 0 and $x+y\leq 1$.
16. If A_r^{pq} and B_t^s are tensors, prove that $C_{rt}^{pqs}=A_r^{pq}B_t^s$ is also a tensor.
17. Compute the divergence and curl of the vector $F=xyzi+3x^2yj+\left(xz^2-y^2z\right)k$ at $(1,2,-1)$.
18. A non-empty subset H of a group G is a subgroup of G if and only if
    (i) $a,b\in H$ implies that $ab\in H$ (ii) $a\in H$ implies that $a^{-1}\in H$.

SECTION C

Answer any **TWO** questions: **(2x20 = 40)**

19. (a) Evaluate $\int \frac{3x+1}{(x-1)^2(x+3)}dx$.

(b)Express $f(x)=\frac{1}{2}(\pi-x)$ as a Fourier series with period 2π, to be valid in the interval 0 to 2π.

And also Deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...=\frac{\pi}{4}$.

(8+12)

20. (a) Solve $\left(D^2-5D+6\right)y=e^{4x}$.

(b) Solve $\left(D^2-4D+3\right)y=\sin 3x\cos 2x$.

(10+10)

21. (a) Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

(b) Find the Jacobian of *x*, *y*, *z* with respect to r, φ,θ where (r, φ,θ) are spherical coordinates.

(15 +5)

22. (a) Evaluate $\iint\left(x^3dydz+x^2ydzdx+x^2zdxdy\right)$ over the surface bounded by z = 0, z = c, $x^2 + y^2 = a^2$ using Green's theorem.

(b) Show that $G=\{1,-1,i,-i\}$ is an abelian group under usual multiplication.

(12 +8)

****