LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

FOURTHSEMESTER – APRIL 2018

MT 4203- ADVANCED MATHEMATICS FOR PHYSICS

Date: 02-05-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

SECTION A

Answer **ALL** the questions: **(10x2 = 20)**

1. Evaluate $\int\left(ax+\frac{b}{x^2}\right)dx$.
2. State Bernoulli's formula.
3. Solve $\frac{dy}{dx}+\left(\frac{1-y^2}{1-x^2}\right)^{1/2}=0$.
4. Write the criterion for $Mdx+Ndy=0$ to be exact.
5. Evaluate $\int_0^3\int_1^2 xy\left(x+y\right)dydx$.
6. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.
7. Prove that $\nabla^2\left(\frac{1}{r}\right)=0$, where *r* is the position vector.
8. State Stokes theorem.
9. Find the Cayley's table for $G=\{1,-1,i,-i\}$ under usual multiplication.
10. Define Contravariant and Covarient tensors.

SECTION B

Answer any **FIVE** questions: **(5x8 = 40)**

11. Evaluate $\int x\sin 2xdx$ using integration by parts method.
12. Show that $\int_0^{\pi/2}\log(\sin x)dx=\frac{\pi}{2}\log\left(\frac{1}{2}\right)$.
13. Solve $\frac{dy}{dx}+y\cos x=\frac{1}{2}\sin 2x$.
14. Solve $\left(D^2-3D+2\right)y=\sin 3x$.
15. Evaluate $\iint xydxdy$ taken over the positive quadrant of the circle $x^2+y^2=a^2$.
16. If $A_r^{p\,q}$ and B_t^s are tensors, prove that $C_{r\,t}^{p\,q\,s}=A_r^{p\,q}\;B_t^s$ is also a tensor.
17. If $F=xy^2i+2x^2yzj-3yzk$, find *div F* and *curl F* at $(1,-1,1)$.
18. Prove that the set $\{1,\omega,\omega^2\}$ is an abelian multiplicative finite group of order 3.

SECTION C

Answer any **TWO** questions: **(2x20 = 40)**

19. (a) Evaluate $\int \frac{6x+5}{\sqrt{6+x-2x^2}}dx$.

(b) Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $(-\pi \le x \le \pi)$.

Deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+...=\frac{\pi^2}{6}$.

(8+12)

20. (a) Solve $(D^2+4D+6)y = 5e^{-2x}$.

(b) Solve $(D^2-3D+2)y = \sin 3x$.

(10+10)

21. (a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

(b) Change the order of integration in the integral $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}dxdy$ and evaluate it.

(14 +6)

22. (a) Find by Green's Theorem the value of $\int_C (x^2 ydx + ydy)$ along the closed curve C formed by $y^2 = x$ and $y = x$ between $(0,0)$ and $(1,1)$.

(b) Show that the union of two subgroups of G is a subgroup if and only if one is contained in the other.

(12 +8)

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