LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2018**

MT 4502/ MT 4500 – MODERN ALGEBRA

Date: 09-05-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

ANSWER ALL QUESTIONS (10 x 2 = 20)

1. Give an example of a one to one onto function.
2. Define partially ordered and totally ordered set.
3. Define cyclic group. Give an example.
4. Define a normal subgroup with an example.
5. Define homomorphism.
6. Define odd and even permutations.
7. Define a field with an example.
8. Define an integral domain.
9. Define maximal ideal.
10. State Unique factorization theorem.

PART – B

ANSWER ANY FIVE QUESTIONS. (5 x 8 = 40)

11. Show that every group of order four is abelian.
12. Prove that there is a one to one correspondence between any two left cosets of a subgroup H in a group G.
13. Show that the intersection of two normal subgroups is again a normal subgroup.
14. Prove that the kernel of a homomorphism f in a group G is a normal subgroup of G.
15. State andprove second isomorphism theorem.
16. Prove that every finite integral domain is a field.
17. Prove that every field is a PID.

18. Let R be a commutative ring with unity and M an ideal of R. Then prove that M is a maximal ideal of R if and only if R/M is a field.

PART – C

ANSWER ANY TWO QUESTIONS. **(2 x 20 = 40)**

19. a. Show that a group G cannot be the union of two proper subgroups. **(10+10)**

b. If H and K are finite subgroups of a group G, then prove that $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$.

20. a. State and prove Lagrange's theorem.

b. State and prove Cayley theorem. **(10+10)**

21. a. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.

b. If p is prime then prove that Z_p is a field. **(10+10)**

22. a. Prove that an ideal of the Euclidean ring R is a maximal ideal of R if and only if it is generated by a prime element of R.

b. Let R be a commutative ring with unity and P an ideal of R. Prove that P is a pime ideal of R if and only if R/P is an integral domain. **(10+10)**

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