LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FIFTHSEMESTER – APRIL 2018

MT 5408- GRAPH THEORY

Date: 08-05-2018   Dept. No.   Max. : 100 Marks
Time: 09:00-12:00

PART – A

Answer ALL questions   **(10 x 2 = 20)**

1. What is isolated vertex? Can you give an example?

2. Prove that $\delta \leq \frac{2q}{p} \leq \Delta$.

3. Show that in any graph G the number of vertices of odd degree is even.

4. Define distance between any two vertices of a graph.

5. Define Hamiltonian graph with an example.

6. What is eccentricity of a graph?

7. Define block with an example.

8. What is planar graph?

9. Show that $K_{3,3}$ is not planar.\

10. Find the chromatic number for the following graph
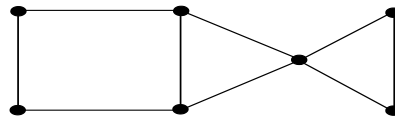
PART – B

Answer any FIVE questions   **(5 x 8 = 40)**

11.Define : a) Parallel edge b) simple graph c) loop

12. Prove that sum of the degree of the vertices of the graph G is twice the number of edges.

13. Define self complementary graph and prove that any self complementary graph has $4n$ or $4n+1$ vertices.

14. If G is a graph in which the degree of every vertex is at least two then prove that G is contains a cycle.

15. Explain Konigsberg Bridge Problem.

16. If G is connected planar graph having V,E and F as the set of vertices, edges and faces respectively then, prove that $|v| - |E| + |F| = 2$.

17. Prove that every non – trivial tree has atleast two vertices of degree one.

18.a) Prove that k_5 is not planar graph

b) In any connected (p, q) planar graph and $p \geq 3$ with r faces, prove that $q \geq \frac{3r}{2}$ and $q \leq 3p - 6$.

PART – C

19.(a) If Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph then prove that
   (i) $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$ graph

   (ii) $G_1 \times G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph

   (b)The maximum number of edges among all p vertex graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

   (6+ 14)

20.(a)Prove that a graph G with P vertices and $\delta \geq \frac{p-1}{2}$ is connected.

   (b) Prove that a graph G with atleast two vertices is bipartite iff all its cycles are of even length. (5 +15)

21.(a) If G is a graph with $p \geq 3$ vertices and $\delta \geq {}^{p}/_{2}$, then prove that G is Hamiltonian.

   (b) Prove that every planar graph is five colourable. (6 + 14)

22.(a) Let G be a (p, q) graph then prove that the following statements are equivalent
   (i) G is a tree.
   (ii) Every two points of G are joined by a unique path.
   (iii)G is connected and $p = q + 1$.
   (iv)G is acyclic and $p = q + 1$.

   (b) If G is a connected (p, q) planar graph with no triangle and $p \geq 3$ then prove that $q \leq 2p - 4$ (14 + 6)

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