LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2018**

MT 5508/ MT 5502 – LINEAR ALGEBRA

Date: 04-05-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART A

ANSWER ALL THE QUESTIONS $(\mathbf{10 * 2 = 20} \boldsymbol{marks})$

1. Define a vector space V over a field F.
2. Express the vector (1,-2,5) as a linear combination of the vectors (1,1,1), (1,2,3) and (2,-1,1) in $\mathcal{R}^3$ where $\mathcal{R}$ is the field of real numbers.
3. Prove that the vectors $(1,0,0), (1,1,0)$ and $(1,1,1)$ form a basis of R^3, where R is the field of real numbers.
4. Define rank and nullity of a vector space homomorphism $T: U \to V$.
5. Define orthonormal set.
6. Let $T \in A(v)$ and $\lambda \in F$. If $\lambda I - T$ is singular then prove that λ is an eigenvalue of T.
7. Define trace of a matrix and give an example.
8. If A is any square matrix, prove that $A + A^t$ is symmetric and $A - A^t$ is a skew-symetric.
9. Find the rank of the matrix A= $\begin{pmatrix} 1 & 5 & -7 \\ 2 & 3 & 1 \end{pmatrix}$ over the field of rational numbers.
10. Define unitary linear transformation.

PART B

ANSWER ANY FIVE QUESTIONS $(\mathbf{5 * 8 = 40} \boldsymbol{marks})$

11. Prove that the union of two subspaces of a vector spaces V over F is a subspace of V if and only if one is contained in the other.
12. Show that a nonempty subset W of a vector space V over F is a subspace of V if and only if $aw_1 + bw_2 \in W for all a, b \in F, w_1, w_2 \in W$.
13. Let V be a vector space and suppose that one basis has n elements and another basis has m elements .Then prove that $m = n$.
14. If A and B are subspaces of a vector space V over F, prove that $(A+B)/B \cong A/A \cap B$.
15. Apply the Gram-Schmidt orthonormalization process to obtain an orthonormal basis for the subspace of R^4 generated by the vectors $(1,1,0,1), (1,-2,0,0), and (1,0,-1,2)$.

16. If $\lambda \in F$ is an eigenvalue of $T \in A(V)$, then prove that for any polynomial $f(x) \in F[x]$, $f(\lambda)$ is an eigenvalue of $f(T)$.

17. Show that any square matrix A can be expressed uniquely as the sum of a symmetric matrix and a skew-symmetric matrix.

18. a) If $T \in A(V)$ is skew-Hermitian, prove that all of its eigenvalues are pure imaginaries.

b)Prove that the eigenvalues of a unitary transformation are all of absolute value 1.

PART C

ANSWERANY TWO QUESTIONS $(\mathbf{2 * 20 = 40} \boldsymbol{marks})$

19. a) Prove that the vector space V over F is a direct sum of two of its subspaces W_1 and W_2 if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = (0)$.

b) If S and T are subsets of a vector space V over F,then prove that

$$L(S \cup T) = L(S) + L(T).$$

20. If V is a vector space of finite dimension and is the direct sum of its subspaces U and W then prove thatdim $V = dimU + \dim W$.

21. State and prove Gram-Schmidt orthonormalization process.

22. a) If $A, B \in F_n$ and if $\lambda \in F$,then prove that

i) $(\lambda A)^t = \lambda A^t$

ii) $(A^t)^t = A$

iii) $(A + B)^t = A^t + B^t$

iv) $(AB)^t = B^t A^t$

b) Investigate for what values of λ, μ the system of equations

$x_1 + x_2 + x_3 = 6, x_1 + 2x_2 + 3x_3 = 10, x_1 + 2x_2 + \lambda x_3 = \mu$ over the rational field has a) no solution b) a unique solution c) an infinite number of solutions.

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