LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2018**

MT 6606– COMPLEX ANALYSIS

Date: 17-04-2018 | Dept. No. | Max. : 100 Marks

Time: 09:00-12:00

PART-A

ANSWER ALL THE QUESTIONS: (10x2=20marks)

1. Verify Cauchy-Riemann equations for the function $f(z) = z^3$.
2. Show that $u = log\sqrt{x^2 + y^2}$ is harmonic.
3. Find the points where the mapping $w = e^z$ is conformal. Also find the critical points.
4. Define a bilinear transformation.
5. Using Cauchy's Integral formula, evaluate $\frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3}dz$ where C is $|z| = 4$.
6. Evaluate $\int_C \frac{e^z}{z^n}dz$ where C is the circle $|z|=1$.
7. Find the poles of $f(z)=\frac{z^2-2z+3}{z-2}$
8. Write Maclaurin's series expansion of $sinz$.
9. Find the residue of $\frac{ze^z}{(z-1)^3}$ at its poles.
10. State Cauchy's residue theorem.

PART-B

ANSWER ANY FIVE QUESTIONS: (5x8=40marks)

11. Prove that $f(z) = \sin x \cosh y + i \cos x \sinh y$ is differentiable at every point.

12. If $f(z)$ is analytic prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\left|f^{'}(z)\right|^2$.

13. Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ onto $w_1 = 1, w_2 = i, w_3 = -1$ respectively.

14. State and prove Liouville's theorem.

15. Expand $f(z)=\frac{z-1}{z+1}$ as a Taylor's series

(i) about the point $z = 0$.

(ii) about the point $z = 1$. Determine the region of convergence in each case.

16. Evaluate $\int_C \frac{3z^2+z-1}{(z^2-1)(z-3)}dz$ where C is $|z| = 2$ by using residue theorem.

17. Evaluate by using Cauchy's integral formula $\int_C \frac{z+1}{z^2+2z+4}dz$ where C is the circle $|z+1+i| = 2$

18. State and prove Rouche's theorem.

PART-C

ANSWER ANY TWO QUESTIONS: (2 x 20=40marks)

19. a) Derive C.R equations in polar coordinates.

b) Prove that any bilinear transformation can be expressed as a product of translation, rotation, magnification or contraction and inversion. **(12+8)**

20. a) State and prove Cauchy's integral theorem.

b) Evaluate $\int_C (\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)})$ where C is the circle $|z| = 3$.**(12+8)**

21. a) State and prove Laurent's theorem.

b) State and prove fundamental theorem of algebra. **(12+8)**

22. a) State and prove Argument theorem.

b) Using the method of contour integration evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}dx$

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