LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2019**

16/17/18PMT1MC03– ORDINARY DIFFERENTIAL EQUATIONS

Date: 08-04-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

ANSWER ALL QUESTIONS

1. (a) State and prove Abel's formula. **(5)**

(OR)

(b) Let $x_p(t)$ be any particular solution of $L[x(t)] = d(t)$ and $x_h(t)$ be the general solution of $L[x(t)] = 0$. Show that $x(t) = x_p(t) + x_h(t)$ is the general solution of $L[x(t)] = d(t)$. **(5)**

(c) Explain the method of variation of parameters for the second order differential equation. **(15)**

(OR)

(d) Derive the various solutions of the second order linear homogenous differential equation with constant coefficients. **(15)**

2. (a) State and prove Rodrigue's formula. **(5)**

(OR)

(b) Prove that (i) $P_l(1) = 1$, and (ii) $P_l'(1) = \frac{1}{2}l(l+1)$. **(5)**

(c) Solve $2x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$by Frobenius method. **(15)**

(OR)

(d) Derive the orthogonality properties of the Legendre's polynomial. **(15)**

3. (a) When *n* is a non-zero integer, show that $J_{-n}(x) = (-1)^n J_n(x)$. **(5)**

(OR)

(b) Derive the generating function for Bessel's function. **(5)**

(c) Let $J_n(x)$ be the Bessel's function of first kind of order *n* and the Bessel's function of second kind be $Y_n(x) = \frac{\cos n\pi\, J_n(x) - J_{-n}(x)}{\sin n\pi}$. Prove that the two independent solutions of the Bessel's equation are$J_n(x)$ and $Y_n(x)$for all values of *n*. **(15)**

(OR)

(d) State and prove the integral representation of Bessel's function. **(15)**

4. (a) Prove that all the eigenvalues of Strum-Liouville problem are real. **(5)**

(OR)

(b) Using the method of successive approximations, solve the initial value problem $x'(t) = -x(t)$, $x(0) = 1, t \geq 0$. **(5)**

(c) Prove that $x(t)$ is a solution of a non-homogeneous equation$L(x(t)) + f(t) = 0, a \leq t \leq b$, if and only if $x(t) = \int_a^b G(t,s)f(s)\,ds$ where $G(t,s)$ is the Green's function. **(15)**

(OR)

(d) State and prove Picard's theorem for initial value problem. **(15)**

5. (a) Show that the null solution of the equation $x' = A(t)x$is stable if and only if there exists a positive constant$k$ such that $|\phi(t)| \leq k, t \geq t_0$. **(5)**

(OR)

(b) Define autonomous system and state it stability behaviors. **(5)**

(c) Discuss the stability of linear system $x' = Ax$ usingLyapunov'sfunction. **(15)**

(OR)

(d) State and prove the two fundamental theorems on the stability of non-autonomous systems. **(15)**

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