LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2019**

16UMT6MC04– GRAPH THEORY

Date: 13-04-2019 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART – A

Answer ALL questions. **(10 ò 2 = 20)**

1. State Königsberg bridge problem.
2. Give examples of two isomorphic graphs.
3. When do you say that a graph is Euler?
4. Define Hamiltonian circuit.
5. In a graph, which vertex is called a pendent vertex?
6. Define spanning tree of a connected graph.
7. What is cut- set of a connected graph?
8. Define an edge connectivity of a graph.
9. Give an example of a non planar graph.
10. Define an asymmetric digraph.

PART – B

Answer any FIVE questions **(5 ò 8 = 40)**

11. What is utility problem and explain how it can be represented by a graph.
12. In a connected graph G with exactly 2k odd vertices, show that there exist k edge disjoint sub graphs such that they together contain all edges of G and that each is a unicursal graph.
13. Show that a connected graph is an Euler graph if and only if it can be decomposed into circuits.
14. Prove that a tree with n vertices has $n-1$ edges.
15. Prove that every circuit has an even number of edges in common with any cut set.
16. Show that edge connectivity of a graph G cannot exceed the degree of the vertex with the smallest degree in G and the vertex connectivity of any graph G can never exceed the edge connectivity of a G.
17. Show that the complete graph of five vertices is non- planar..
18. A graph of n vertices is a complete graph if and only if its chromatic polynomial is $P_n(\lambda) = \lambda (\lambda -1) (\lambda -2) \ldots\ldots (\lambda - n+1)$.

PART – C

Answer any TWO questions. **(2 ò 20 =40)**

19. (a) Show that a simple graph with n vertices and k components can have at most (n-k)(n-k+1)/2 edges.

(10)

(b) Show that a connected graph is an Euler graph if and only if all vertices of G are of even degree.

(10)

20. (a) Show that in a complete graph with n vertices there are (n-1)/2 edge disjoint Hamiltonian circuits, if n is an odd number ≥ 3. **(10)**

(b) Show that every tree has either one or two centers. **(10)**

21. (a) Prove that a spanning tree of a given weighted connected graph G is a shortest spanning tree of G if and only if there exists no other spanning tree of G at a distance of one from T whose weight is smaller than that of T. **(10)**

(b) Show that with respect to a given spanning tree T, a branch b_i that determines a fundamental cut set S is contained in every fundamental circuit associated with the chords in S, and in no others.

(10)

22. (a) Prove that a connected planar graph with n vertices and e edge has e-n+2 regions**. (10)**

(b) Show that every tree with two or more vertices is 2- chromatic. **(10)**

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