LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2019**

MT 2502– ALGEBRA AND CALCULUS - II

Date: 02-04-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

ANSWER ALL QUESTIONS $(\mathbf{10X2 = 20})$

1. Evaluate $\int_0^{\frac{\pi}{2}} \cos^2 \frac{x}{2}\, dx$.
2. Evaluate $\int \tan^4 x dx$.
3. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} d\theta d\varphi$.
4. Find $\frac{\partial(x,y)}{\partial(r,\theta)}$, if $x = r\cos\theta\,, y = r\sin\theta$.
5. Define Beta and Gamma functions.
6. Prove that $\beta(m,n) = \beta(n,m)$.
7. Define Comparison test
8. Test the convergence of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$.
9. Expand $(1+x)^{\frac{p}{q}}$.
10. Find the coefficient of x^n in the expansion of $\frac{1+2x-3x^2}{e^x}$.

PART – B

ANSWER ANY FIVE QUESTIONS $(\mathbf{5X8 = 40})$

11. Prove that $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \frac{\pi}{4}$.
12. Change the order of integration and hence evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx$.
13. Express $\int_0^1 x^m (1-x^n)^p dx$ in terms of Gamma functions.
14. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
15. Show that the series $1 + \frac{2^2}{2!} + \frac{3^3}{3!} + \frac{4^4}{4!} + \cdots$ is divergent.
16. Test the convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\log n)^2}$.
17. Sum the series $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \cdots \infty$.
18. Show that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)2n(2n+1)} = \log 2 - \frac{1}{2}$.

PART – C

ANSWER ANY TWO QUESTIONS **($2X20 = 40$)**

19. (a) Find the length of one loop of the curve $3ay^2 = x(x-a)^2$.

    (b) Evaluate $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$. (10 + 10)

20. Find the area of the cardioids $r = a(1 + \cos\theta)$.

21. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(n+n)}$.

22. (a) Sum the series to infinity $\frac{1.4}{5.10} - \frac{1.4.7}{5.10.15} + \frac{1.4.7.10}{5.10.15.20} - \cdots$.

    (b) Show that $\log x = \frac{x-1}{x+1} + \frac{1}{2}.\frac{x^2-1}{(x+1)^2} + \frac{1}{3}.\frac{x^3-1}{(x+1)^3} + \cdots$, if $x > 0$. (10 + 10)

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