LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION –**MATHEMATICS**

SECOND SEMESTER – **APRIL 2019**

MT 2503– ANALY. GEOM. OF 3D, FOURIER SERIES & NUM. THEORY

Date: 04-04-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART-A **(10× 2 = 20)**

Answer all questions:

[1]Find the distance of the origin from the plane $6x - 3y + 2z - 14 = 0$.

[2]Find the equation of the plane through (1, 1,1) and the line of intersection of the planes $x + 2y - z + 1 = 0$;$3x - y + 4z - 3 = 0$.

[3]Find the equationof the sphere with centre($-1 , 2 , -3$) and radius 3 units.

[4]Find the equation of the tangent plane to the sphere $(x^2 + y^2 + z^2) - 4x - 4y - 2z - 22 = 0$ at the point (2 , 3 , 1) .

[5] State two properties of even and odd periodic functions.

[6] State Dirichlet's conditions.

[7] Find the number of divisors of 720.

[8] Findϕ(729).

[9]Show that $n^n > 1.3.5 \ldots .(2n - 1)$.

[10]State Weirstrass inequalities.

PART-B **(5× 8 = 40)**

Answer any FIVE

[11]Find the equation of the plane passing through the points (3 , 1 ,2)(3 , 4 , 4) and perpendicular to the plane $5x + y + 4z = 0$.

[12] Find the image of the point (1 , -2 , 3) in the plane $2x - 3y + 2z + 3 = 0$.

[13]Find the equation to a sphere through the four points (2 , 3 , 1)(5 , -1 , 2)(4 , 3 , -1)and (2 , 5 , 3) .

[14]Find the equation of a sphere which touches the sphere$x^2 + y^2 + z^2 - 6x + 2z + 1 = 0$ at the point(2 , -2 , 1) and passes through the origin.

[15] Find the Fourier series for $f(x) = x , -\pi \leq x \leq \pi$.

[16] Find the smallest number with 18 divisors.

[17] If x and y are positive quantities whose sum is 4 ,show that $(x + \frac{1}{x})^2 + (y + \frac{1}{y})^2 \nless 12\frac{1}{2}$.

[18]Show that if a, b, c are three positive unequal quantities , then$\frac{a^8+b^8+c^8}{a^3b^3c^3} > \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

PART-C **(2× 20 = 40)**

Answer any TWO:

[19] (i) Prove that equation of the first degree in x , y, z represents a plane. (10 + 10)

(ii) Prove that the lines$\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$; $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$are coplanar.

Find also their point of contact and the plane through them.

[20] (i) A plane passes through a fixed point (a, b, c) and cuts the axes in A,B,C.

Show that the locus of the centre of the sphere OABC is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$. (10+10)

(ii)Show that the plane $2x-y-2z=16$ touches the sphere

$x^2+y^2+z^2-4x+2y+2z-3=0$ and find the point of contact.

[21] (i)Show that $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2}$ in the interval $-\pi\le x\le\pi$.(12+8)

Deduce that (a) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\cdots=\frac{\pi^2}{12}$.

(b) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}$.

(c) $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}$.

(ii)Find the Fourier series for $f(x)=\frac{1}{2}(\pi-x)$ in the interval 0 to 2π.

[22](i)With how many zeros does 79! Ends? (5+5+10)

(ii) Find the remainder obtained in dividing 2^{46} by 47.

(iii) Show that $(x^m+y^m)^n<(x^n+y^n)^m$ if $m>n$.

****