



Date: 08-04-2019
Time: 09:00-12:00

Dept. No.

Max. : 100 Marks

ANSWER ALL QUESTIONS

1. (a) Define adjacency matrix and give an example.

[OR]

(b) Show that any 3-regular has even number of points.

(5)

(c) (i) Show that a graph is bipartite if and only if it contains no odd cycle.

(ii) Show that an edge e is a cut edge of G if and only if e is contained in no cycle of G

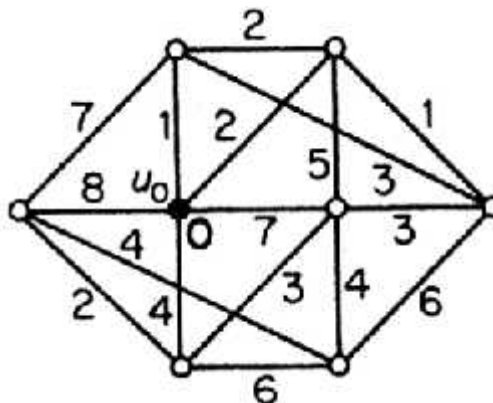
(10+5)

[OR]

(d) (i) Show that every connected graph contains a spanning tree.

(ii) Apply Dijkstra's algorithm to find shortest path from u_0 to all other vertices of the following graph.

(5+10)



2. (a) State and prove Kuraskal's algorithm

[OR]

(b) Define Hamiltonian cycles. Construct a Hamiltonian cycle in a dodecahedron.

(c) (i) Define closure of a graph G . Prove that closure of a graph is well defined.

(ii) Show that a simple graph is Hamiltonian if and only if its closure is hamiltinian.

(10+5)

[OR]

(d) (i) State and prove Dirac theorem for Hamiltonian graphs.

(ii) Prove that a nonempty connected graph is eulerian if and only if it has no vertices of odd degree.

(10+5)

3. (a) Define perfect matching and give an example. **[OR]**
- (b) Find the number of perfect matchings of K_{2n} . **(5)**
- (c) (i) State and prove Berge's theorem for matchings.
(ii) Show that a tree has at most one perfect matching. **(10+5)**
- [OR]**
- (d) (i) State and prove Hall's theorem.
(ii) Show that every 3-regular graph without cut edges has a perfect matching. **(12 + 3)**

4. (a) Define independent set and give an example. **[OR]**
- (b) Prove every critical graph is a block. **(5)**
- (c) State and prove Dirac theorem for critical graphs in vertex coloring. **(15)**
- [OR]**
- (d) (i) In a critical graph show that no vertex cut is a clique.
(ii) Write the chromatic polynomial for the following graphs



(8+7)

5. (a) Prove that K_5 is non planar. **[OR]**
- (b) Define bridges and give an example. **(5)**
- (c) State and prove Kuratowski's theorem. **(15)**
- [OR]**
- (d) (i) State and prove five color theorem.
(ii) Show that inner bridges avoid one another. **(10+5)**

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