LOYOLA COLLEGE (AUTONOMOUS) CHENNAI-600 034
B.Sc. DEGREE EXAMINATION - STATISTICS
SECOND SEMESTER – APRIL 2019
STA 501- STATISTICAL MATHEMATICS - I

Max:100 Marks

PART – A

Answer any FOUR questions **(4 X 10 = 40 Marks)**

1. Using the definition of limits prove that $\lim_{x\to a}\frac{x^2-a^2}{x-a}=2a$.
2. If $f(x)=\frac{x^2-9}{x-3}$, check whether $f(x)$ is continuous at a point 3.
3. Show that every convergent sequence is bounded. Is the converse true?
4. Prove that a monotonic increasing sequence which is bounded above is convergent.
5. Establish the convergence of $\sum_{n=1}^{\infty}\frac{1}{n^2}$.
6. State and prove Lagrange's Mean Value theorem.
7. Find the inverse of a matrix $\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$
8. A continuous random variable X with pdf $f(x)=k(x-x^2), 0\le x\le 1$. Determine the constant k and E(X).

PART –B

Answer any THREE questions **(3 X 20 = 60 Marks)**

9. Two random variables X and Y have the joint pdf given by

$$f(x,y)=\begin{cases}6x^2y, 0<x<1, 0<y<1\\ 0, \quad elsewhere\end{cases}$$

i) Verify that $\int_0^1\int_0^1 f(x,y)dxdy=1$

ii) Find P (X + Y < 1).

10. State and prove Rolle's theorem.
11. Verify whether the vectors (2, 5, 3), (1, 1, 1) and (4, -2, 0) are linearly independent.
12. Find the rank of $\begin{bmatrix}1 & 2 & 3-2\\ 2 & -2 & 1\ 3\\ 3 & 0 & 4\ 1\end{bmatrix}$.

13. An experiment consists of 3 independent tosses of a fair coin. Let X be the number of heads, Y be the number of head runs, Z be the length of head runs. A head run being defined as consecutive occurrence of at least two heads, its length then being the number of heads occurring together in three tosses of the coin. Find the probability function of i) X, ii) Y, iii) Z .

14. If the joint distribution function of X and Y is given by

$$F(x\ y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-(x+y)} ; x > 0, y > 0 \\ 0 \qquad\qquad\qquad ; otherwise \end{cases}$$

a) Find the marginal densities of X and Y.

b) Are X and Y independent?