LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2019**

MT 5509– ALGEBRAIC STRUCTURE - II

Date: 22-04-2019 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART –A

ANSWER ALL THE QUESTIONS: **(10 X 2 = 20)**

1. Define linear span.
2. If V is a vector space over a field F, show that $(-a)v = a(-v) = -av \; for a \epsilon F, v \epsilon v \epsilon V$.
3. Find the coordinate vector of (2,1,-6) of R^3 relative to the basis $\{(1,1,2),(3,-1,0),(2,0,-1)\}$.
4. Define eigen value and eigen vector of the matrix.
5. Define inner product spaces.
6. Define orthonormal set.
7. Show that the matrix A = $\begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$ is orthogonal.
8. Define trace of A and give an example.
9. Define unitary linear transformation.
10. Find the rank of matrix $\begin{pmatrix} 2 & 0 \\ 1 & 1 \\ 1 & -1 \end{pmatrix}$.

PART - B

ANSWER ANY FIVE QUESTIONS: **(5 X 8 = 40)**

11. Prove that the intersection of two subspaces of a vector space V is a subspace of V.

12. If S and T are subsets of a vector space V over F, then prove that

    (i) S is a subspace of V if and only if L(S) = S.

    (ii) S $\subseteq$ T implies that L(S) $\subseteq$ L(T).

    (iii) L(L(S)) = L (S).

13. Let V and W be two n-dimensional vector spaces over F, then prove that any isomorphism T of V onto W maps a basis of V onto a basis of W.

14.State and prove Schwarz inequality.

15.If $\lambda \epsilon F$ is an eigenvalue of $T \in A(V)$, then prove any polynomial $f(x) \in F[x]$, $F(\lambda)$ is an eigenvalue of $f(T)$.

16. If $A, B \in F_n$, and $\lambda \in F$,then prove that

(i) $tr(\lambda A) = \lambda trA.$

(ii) $tr(A + B) = trA + trB.$

(iii) $tr(AB) = tr(BA).$

17.Show that any square matrix A can be expressed uniquely as the sum of a symmetric matrix and a skew-symmetric matrix.

18.Solve the system of linear equations

$x_1 + 2x_2 + 2x_3 = 5,\quad x_1 - 3x_2 + 2x_3 = -5,\ 2x_1 - x_2 + x_3 = -3.$

over the rational field.

PART - C

ANSWER ANY TWO QUESTIONS: **(2 X 20 = 40)**

19.(a) The vector space V over F is a direct sum of two of its subspace $W_1 and\, W_2$ if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = (0)$.

(b) If V is a vector space over F, then prove that

(i) $a0 = 0$ for$a \in F$ (ii) $0v = 0$ for $v \in V$

20. If $V$is a vector space of finite dimension and W is a subspace of V, then prove that $dim\frac{V}{W} = dimV - dimW$.

21.Prove that every finite-dimensional inner product space V has an orthonormal set as a basis.

22. (a) If A and B are Hermitian ,show that $AB + BA$ is Hermitian and $AB - BA$ is skew-Hermitian.

(b) If$< T(v), T(v) >=< v, v >$ for all v in$V$, then prove that T is unitary.

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