LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2022**

16/17/18UMT4MC01 – ABSTRACT ALGEBRA

Date: 16-06-2022 Dept. No. Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL Questions: **(10 × 2 = 20)**

1. Show that if every element of a group G is its own inverse, then G is abelian.
2. For all $a, b \in G$, show that $(a\,.\,b)^{-1} = b^{-1}a^{-1}$.
3. Define quotient group.
4. Show that the intersection of two normal subgroups of G is also a normal subgroup of G.
5. Let $G = \{1, -1, i, -i\}$ be the group under multiplication and I be group of all integers under addition. Prove that the mapping $f: I \to G$ such that $f(x) = i^n \; \forall n \in I$, is a homomorphism.
6. Write the cycles of $\begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8 \\ 3\,5\,2\,4\,7\,8\,1\,6 \end{pmatrix}$.
7. Show that kernel of a ring homomorphism is an ideal.
8. When is an integral domain said to be of characteristic zero?
9. Define Euclidean ring.
10. Find all units of J[i].

PART – B

Answer any FIVE Questions: **(5 × 8 = 40)**

11. If G is group of even order, prove that it has an element $a \neq e$ satisfying $a^2 = e$.
12. State and prove the necessary and sufficient condition for a nonempty subset of a group to be a subgroup of the group.
13. Show that the subgroup N of a group G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.
14. If *H* and *K* are subgroups of a group *G*, prove that *HK* is a subgroup of *G* if and only if *HK=KH*.
15. State and prove Cayley's theorem.

16. If ϕ is a homomorphism of a group G into another group G with kernel *K*, prove that *K* is a normal subgroup of G.

17. If *U* is an ideal of a ring *R* show that *R/U* is also a ring.

18. Let *R* be a Euclidean ring. Show that any two elements *a* and *b* in *R* have a greatest common divisor *d* which can be expressed as λa + μb for some λ, μ in *R*.

PART − C

Answer any TWO Questions: **(2 × 20 = 40)**

19. (a) If G is a group, show that for all $a \in G, H_a = \{x \in G, a \equiv x \; mod \; H\}$. (8)

(b) Show that any group of prime order is cyclic. (7)

(C) If G is a group, show that the set of all automorphisms $\mathcal{A}(G)$ of G is also a group. (5)

20. (a) If H and K are finite subgroups of G, then show that $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$. (11)

(b) If ϕ is a homomorphism of a group G onto a group $\bar{G}$ with kernel *K*, then prove that $G/K \equiv \bar{G}$. . (9)

21. (a) Show that the alternating group A_n is a normal subgroup of the symmetric group S_n of index two. (6)

(b) Prove that a finite integral domain is a field. (14)

22. (a) If U is an ideal of a ring R show that R/U is also a ring. (3)

(b) Show that J[i] is a Euclidean ring. (10)

(c) State and prove unique factorization theorem. (7)

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