LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2022**

16/17/18UMT6MC03 – DISCRETE MATHEMATICS

Date: 20-06-2022 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

Answer ALL questions **(10x2= 20)**

1. What is a declarative sentence?
2. Define monoid with an example.
3. Prove that in a lattice if $a \leq b$ then $a \oplus b = b$.
4. How do you justify the consistency of given any two premises?
5. Write the following statement in symbolic form, 'Moscow is neither a country nor a state'.
6. Define semigroup homomorphism.
7. When do you say an element to be join-irreducible?
8. State the rules of inference.
9. Discuss the conditions for a Boolean expression to be symmetric.
10. What is a complemented lattice?

PART – B

Answer any FIVE questions **(5 x 8 = 40)**

11. What is an idempotent element? Prove that for any commutative monoid (M, $*$), the set of idempotent elements of M forms a submonoid.
12. Construct the truth table of $(i)\ \big(Q \wedge (P \to Q)\big) \to P$ $(ii)\ \neg(P \wedge Q) \rightleftarrows (\neg P \vee Q)$.
13. (a) Show that $P(x) \wedge (x)Q(x) \Rightarrow (\exists\, x)\,(P(x) \wedge Q(x))$.

    (b) Prove that the conclusion $R \vee S$ follows from the premises $(C \vee D) \to \neg H$, $\neg H \to (A \wedge \neg B)$ and $(A \wedge \neg B) \to (R \vee S)$ using equivalence laws.
14. Show that in a complemented distributive lattice $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$.
15. Prove that the quotient set $(S/R, \oplus)$ is a semigroup, where R is congruence relation defined on a semigroup (S,*). Also verify whether there exists a natural homomorphism from (S,*) onto $(S/R, \oplus)$.
16. Express the following sentences in symbolic form using the corresponding quantifiers. (i) All men are giants. (ii) Integers are either positive or negative. (iii) X is the father of mother of Y. (iv) Some cats are black.

17. State and prove Stone's Representation theorem.

18. Define least upper bound and greatest lower bound and prove that every finite lattice is bounded.

PART – C

Answer any TWO questions **(2 x 20 = 40)**

19. (a) Express the following Boolean expressions in an equivalent sum of the product of canonical forms in three variables x_1, x_2 and x_3 (i) $x_1 * x_2$. (ii) $x_1 \oplus x_2$. $(iii)(x_1 \oplus x_2)' * x_3$.

(b) Obtain the principal disjunctive and conjunctive normal forms of

(Q → P) ∧ (˥P ∧ Q). (10 + 10)

20. (a) State and prove De Morgan's laws of Lattices.

(b) Show that the composition of semigroup homomorphism is also a semigroup homomorphism. (10 + 10)

21. (a) Let X be a set containing n elements, let X^* denote the free semigroup generated by X, and let $(S, \oplus)$ be any other semigroup generated by any n generators then show that there exists a homomorphism $g: X^* \to S$.

(b) Show that the formula $Q \vee (P \wedge 1\ Q) \vee (1\ P \wedge 1\ Q)$ is a tautology with reasons. (10 + 10)

22. (a) Prove that (S_{36}, D) the set of all divisors of 36 and D denote the relation of division is a lattice. Also evaluate the diagrams for S_n; $n = 12,8$.

(b) Verify using rules of inference whether $S \vee R$ is tautologically implied by

$(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$. (10 + 10)

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