LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2022**

PMT 4504 – CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

Date: 20-06-2022 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Answer ALL questions

1. a) Discuss about any two types of kernels with examples.

(OR)

b) Show that $(1+x^2)^{\frac{-3}{2}}$ is solution of $y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t)\, dt.$ **(5)**

c) Convert $y'' - \sin x\, y' + e^x y = x$ with initial condition $y(0) = 1,\ y'(0) = -1$ to Volterra integral equation of the second kind. Conversely, derive the original differential equation with the initial condition from the integral equation obtained. **(15)**

(OR)

d) i) If $y(x)$ is continuous and satisfies $y(x) = \lambda \int_0^1 K(x,t) y(t) dt$, where

$K(x,t) = \begin{cases} (1-t)x, & 0 \le x \le t \\ (1-x)t, & t \le x \le 1. \end{cases}$ then prove that $y(x)$ is also the solution of the boundary value problem $\frac{d^2y}{dx^2} + \lambda y = 0.$ $y(0) = 0,\ y(1) = 0.$

ii) Write about various types of integral equations **(10 + 5)**

2 a) Solve the homogeneous Fredholm equation $y(x) = \lambda \int_0^1 e^x e^t\, y(t)\, dt.$

(OR)

b) Solve: $y(x) = e^x + \lambda \int_0^1 2e^x e^t\, y(t)\, dt.$ **(5)**

c) Determine the eigenvalues and eigenfunctions of the homogeneous integral equation

$y(x) = \lambda \int_0^1 K(x,t)\, y(t)\, dt$ where $K(x,t) = \begin{cases} (x+1)t, & 0 \le x \le t \\ (t+1)x, & t \le x \le 1. \end{cases}$

(OR)

d) Discuss about the solution of Fredholm integral equation of the second kind with separable kernel. **(15)**

3 a) Write the procedure to write the solution Volterra integral equation using reslovent kernel.

(OR)

b) Solve $y(x) = x + \int_0^{1/2} y(t)\, dt$ using resolvent kernel. **(5)**

c) Solve by the method of successive approximation:

$$y(x) = \frac{3}{2}e^x - \frac{1}{2}\,xe^x - \frac{1}{2} + \frac{1}{2}\int_0^1 t\ y(t)\,dt.$$

(OR)

d) Write the solution of Voltera integral equation of the second kind by successive approximations using Neumann series method. **(15)**

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4 a) Show that the shortest distance between two fixed points in the Euclidean plane is a straight line.

(OR)

b) Discuss about special cases of Euler's Equation. **(5)**

c) State and prove Euler's Equations. Also derive the second and third forms. **(15)**

(OR)

d) i) State and prove Brachistochrone Problem.

ii) Prove that the extremal of $\int_{x_1}^{x_2} y(1 + (y')^2)^{1/2}\,dx$ is a catenary. **(8 + 7)**

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5 a) Discuss about proper field with an example.

(OR)

b) Discuss about pencil of curves with an example. **(5)**

c) Find the shortest distance between $P_1(1,0)$ and the ellipse $4x^2+9y^2 = 36$. **(15)**

(OR)

d) i) Find the extremal for $J(y) = \int_1^2 (y'^2 + 2yy' + y^2)dx$ where $y(1) = 1$ and $y(2)$ is arbitrary.

ii) Investigate for an extremum of the functional $I = \int_0^1 (x + 2y + \frac{y'^2}{2})\,dx;\ y(0) = 0, y(1) = 0.$

(8 + 7)

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