LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FOURTH SEMESTER – **APRIL 2022**

UMT 4403 – MATHEMATICS FOR STATISTICS - II

Date: 27-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

Part A

Answer ALL questions **(10 x 2 = 20)**

1. Define monotone sequences.
2. Define greatest lower bound for a set and give an example.
3. What do you mean by a series of real numbers dominated by another series of real numbers?
4. State D'Alembert's ratio test.
5. Define right-hand limit of f at c, give an example.
6. Define strictly increasing and strictly decreasing functions.
7. Using the definition of differentiation, find the derivative of $f(x) = x^3$.
8. State Fundamental Theorem of Calculus.
9. Define sets of measure zero.
10. Using the properties of Riemann integral, evaluate $\int_0^1 (2x^2 - 3x + 5)dx$.

Part B

Answer any FIVE questions **(5 x 8 = 40)**

11. If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent to L, then prove that $\{s_n\}_{n=1}^{\infty}$ cannot also converge to a limit distinct from L.
12. (i) Prove that $\lim_{n\to\infty} \frac{2n^3+5n}{4n^3+n^2} = 2$.

    (ii) Prove that the series $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)$ is divergent. **(4+4)**
13. If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then prove that $\lim_{n\to\infty} a_n = 0$.
14. State and prove Comparison test for absolute convergence.
15. Write briefly about the application of continuous function in Probability Theory and Statistics.
16. Prove that every differentiable function is continuous.
17. Let f be a continuous real-valued function on the closed bounded interval [a, b]. If the maximum value for f attained at c where $a < x < b$, and if $f'(c)$ exists, then prove that $f'(c) = 0$.
18. If $f \in \mathcal{R}[a,b]$, then prove that $|f| \in \mathcal{R}[a,b]$ and $\left|\int_a^b f\right| \leq \int_a^b |f|$.

Part C

Answer any TWO questions **(2 x 20 = 40)**

19. (a) For any three sets A, B and C, prove that $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.

(b) If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers converges to L and M respectively, then prove the following: $(i) \lim_{n\to\infty}(s_n + t_n) = L + M$ $(ii) \lim_{n\to\infty}(s_n t_n) = LM$ $(iii) \lim_{n\to\infty}(s_n^2) = L^2$.

(8+12)

20. (a) Using D'Alembert's ratio test, test the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{x^n}{n}\right)$.

(b) Prove that the sum, difference and product of continuous functions are continuous. **(8+12)**

21. State and prove Taylor's Theorem.

22. Suppose that f and g are in $\Re[a, b]$ then prove that the function $f + g$ is in $\Re[a, b]$ and $\int_a^b (f + g) = \int_a^b f + \int_a^b g$.

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