LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2022**

UMT 5501 – REAL ANALYSIS - II

Date: 15-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

Part - A

Answer ALL questions **(10 x 2 = 20)**

1. Using the definition of limit, prove that $\lim_{x\to c} x = c$.
2. Evaluate: $\lim_{x\to 2}\left(\frac{x^3-7x}{4x^2-5x}\right)$.
3. Define continuity of a function.
4. Write the conditions for Lipschitz function.
5. Using the definition of differentiation, find the derivative of $f(x) = x^2$.
6. Using L'Hospital's rule, evaluate $\lim_{x\to 0}\left(\frac{1-cosx}{x^2}\right)$.
7. Define tagged partition.
8. Define Reimann integrable function.
9. Define compact set.
10. What is meant by Cantor set?

Part - B

Answer any FIVE questions **(5 x 8 = 40)**

11. If p and q are polynomial functions on $\mathbb{R}$ and if $q(c) \neq 0$, then prove that $\lim_{x\to c}\frac{p(x)}{q(x)} = \frac{p(c)}{q(c)}$.
12. Using squeeze theorem of limit, prove that $\lim_{x\to 0}\left(\frac{cosx-1}{x}\right) = 0$.
13. State and prove Rolle's theorem.
14. Let $A \subseteq \mathbb{R}$, let f and g be functions on A to R and let $b \in \mathbb{R}$. If f and g are continuous at c, then prove that $f+g$, fg and bf are also continuous at c.
15. State and prove the location of roots theorem.
16. If $f: [a,b] \to \mathbb{R}$ is monotone on [a, b], then prove that $f \in \mathcal{R}[a,b]$.
17. State and prove Cauchy Criterion for integration.
18. Prove that the union of an arbitrary collection of open subsets in $\mathbb{R}$ is open in $\mathbb{R}$.

Part - C

Answer any TWO questions **(2 x 20 = 40)**

19. State and prove sequential criterion theorem for limits.

20. State and prove the Maximum-Minimum theorem.

21. State and prove Taylor's theorem.

22. (a) Prove that a subset of $\mathbb{R}$ is closed if and only if it contains all its cluster points.

    (b) State and prove fundamental theorem of Calculus. **(10+10)**

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