LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2023**

PMT1MC03 – ORDINARY DIFFERENTIAL EQUATIONS

Date: 03-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

	SECTION A		
	Answer ALL the questions		
1	**Answer the following.** **(5 x 1 = 5)**		
a)	Describe the first order initial value problem.	K1	CO1
b)	Define linear independence.	K1	CO1
c)	When do you say that a matrix corresponds to a linear system is fundamental?	K1	CO1
d)	Define ordinary point.	K1	CO1
e)	Describe the oscillatory differential equation.	K1	CO1
2	**Choose the correct answer.** **(5 x 1 = 5)**		
a)	The first approximate solution of $x' = x^2$, $x(0) = 1$, as per Picard's successive approximation method is (a) 1 (b) $1+t$ (c) $1-t$ (d) t^2	K2	CO1
b)	The Wronskian of e^t and e^{-t} is (a) 1 (b) -1 (c) 2 (d) -2	K2	CO1
c)	When a linear equation $x''' - 4x'' + 10x' - 2x = 0$ is transformed to linear system $x' = Ax$, where A is (a) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -10 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -10 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 2 & -5 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 1 & -5 & 2 \end{bmatrix}$	K2	CO1
d)	Which of the following is not a regular singular point of the equation $t(t-1)^2(t+3)x'' + t^2x' - (t^2+t-1)x = 0$? (a) 1 (b) 0 (c) -2 (d) none of these	K2	CO1
e)	The equation $x'' + x = 0$ is (a) oscillatory (b) non-oscillatory (c) neither (a) nor (b) (d) both (a) and (b)	K2	CO1
	SECTION B		
	Answer any THREE of the following. **(3 x 10 = 30)**		
3	Apply the method of variation of parameters to solve $x' + a(t)x = b(t)$ where a and b are known continuous function defined on the interval I.	K3	CO2
4	Let $b_1, b_2, \dots, b_n : I \to \mathbb{R}$ be continuous functions in the n-th order homogeneous differential equation $L(x) = 0$. Let $\varphi_1, \varphi_2, \dots, \varphi_n$ be n linearly independent solutions of $L(x) = 0$ on I. Calculate the Wronskian of $\varphi_1, \varphi_2, \dots, \varphi_n$.	K3	CO2
5	Consider a linear system $x' = A(t)x$ where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$. Show that $\Phi(\mathrm{t}) = \begin{bmatrix} e^{-3t} & te^{-3t} & t^2e^{-3t}/2 \\ 0 & e^{-3t} & te^{-3t} \\ 0 & 0 & e^{-3t} \end{bmatrix}$ is a fundamental matrix.	K3	CO2
6	Apply the generating function of Bessel to show $J_n(t) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - t\sin\theta)\, d\theta$.	K3	CO2

7	Prove the Sturm's comparison theorem.	K3	CO2
	SECTION C		
	Answer any TWO of the following. (2 x 12.5 = 25)		
8	Point out a general criteria to ensure the Lipschitz condition with supportive examples.	K4	CO3
9	Analyze the various solutions of second order differential equation with constant coefficients.	K4	CO3
10	Derive the formula $P_n(t) = \frac{1}{2^n n!}\frac{d^n}{dt^n}(t^2-1)^n$ and hence prove that $\int_{-1}^{1} P_n(t)\, P_m(t)\, dt = 0$ provided $m \neq n$.	K4	CO3
11	Explain the Hille-Wintner comparison theorem.	K4	CO3
	SECTION D		
	Answer any ONE of the following. (1 x 15 = 15)		
12	Let $x' = A(t)x$ be a linear system where $A: I \to M_n(R)$ is continuous. Suppose a matrix Φ satisfies the system, evaluate $(\det \Phi)'$ and assess that if Φ is a fundamental matrix if and only if $\det \Phi \neq 0$.	K5	CO4
13	Evaluate the linearly independent solutions of Legendre equation.	K5	CO4
	SECTION E		
	Answer any ONE of the following. (1 x 20 = 20)		
14	Prepare the conditions for the existence of a unique solution for the first order equation $x' = f(t,x)$, $x(t_0) = x_0$ and validate for the function $f(t,x) = t - x^2$, $x_0 = 0$.	K6	CO5
15	Suppose there are two living species which depend for their survival on a common source of food supply. Develop a mathematical model to describe this phenomenon and discuss the usefulness of the model to the extinct of any one of the species.	K6	CO5

$$$$$$$