LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2023**

PMT2MC03 – PARTIAL DIFFERENTIAL EQUATIONS

Date: 04-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A – K1 (CO1)

	Answer ALL the questions	**(5 x 1 = 5)**
1.	**Answer the following**	
a)	When do you say that the linear differential operator $F(D, D')$ is reducible?	
b)	Write the Laplace equation in polar coordinates.	
c)	State the maximum-minimum principle for heat equation.	
d)	Write down the one-dimensional wave equation.	
e)	What is the method of images?	

SECTION A – K2 (CO1)

	Answer ALL the questions	**(5 x 1 = 5)**
2.	**Choose the correct answer**	
a)	The auxiliary equation of partial differential equation $Pp + Qq = R$ is (a) $\frac{dx}{R} = \frac{dy}{Q} = \frac{dz}{P}$ (b) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ (c) $\frac{dx}{Q} = \frac{dy}{R} = \frac{dz}{P}$ (d) none	
b)	The boundary conditions $u(x, b) = 0, u(a, y) = 0, u(0, y) = 0, u(x, 0) = f(x)$ due to (a) Dirichlet (b) Neumann (c) Euler (d) Gauss	
c)	The Fourier heat conduction equation (a) $\frac{\partial T}{\partial t} = \frac{1}{\alpha}\nabla^2 T$ (b) $\frac{\partial T}{\partial t} = \alpha\nabla^2 T$ (c) $\frac{\partial^2 T}{\partial t^2} = \frac{1}{\alpha}\nabla^2 T$ (d) $\frac{\partial^2 T}{\partial t^2} = \alpha\nabla^2 T$	
d)	Television and radio waves are the examples of ---------. (a) mechanical waves (b) transverse waves (c) longitudinal waves (d) electromagnetic waves	
e)	The value of the Green function on the boundary (a) 0 (b) 1 (c) -1 (d) none	

SECTION B – K3 (CO2)

	Answer any THREE of the following	**(3 x 10 = 30)**
3.	Apply the Lagrange's method to solve the equation $\begin{vmatrix} x & y & z \\ \alpha & \beta & \gamma \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} & -1 \end{vmatrix} = 0.$	
4.	Solve the equation $(D^2 + 2DD' + D'^2 - 2D - 2D')u = \sin(x + 2y)$.	
5.	Use the separation of variables method to find the solution of Laplace equation in cylindrical coordinates.	
6.	Prove that the solution to the wave equation is unique.	
7.	Show that Green's function has the symmetric property.	

	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8.	Classify and solve the following equation: $u_{xx} - 2sinx\, u_{xy} - cos^2x\, u_{yy} - cosx\, u_y = 0$.
9.	Consider the equation $\nabla^2 u = 0$ over a rectangle of length a and breadth b under the boundary conditions $u_x(0, y) = 0, u_x(a, y) = 0, u_y(x, 0) = 0, u_y(x, b) = f(x)$. Analyze the solution $u(x, y)$.
10.	Compare the solutions of the one-dimensional wave equation by canonical reduction and D'Alembert's method.
11.	Identify the Green's function for the Dirichlet's boundary value problem $\nabla^2 u = f$ valid in certain region $\mathcal{R}$ subject to the boundary condition $u = g$ on $\partial\mathcal{R}$.
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12.	Consider the equation $\nabla^2 u = 0$ over a circle of radius a. Evaluate u under the boundary condition $u(a, \theta) = f(\theta)$.
13.	Prove that the following partial differential equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and determine their common solution.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14.	The ends A and B of a rod 10 cm in length are kept at temperature 0°C and 100°C until the steady state condition prevails. Suddenly the temperature at the end A is increased to 20°C and the end B is decreased to 60°C. Formulate the temperature distribution in the rod at time t.
15.	Let a thin homogeneous string that is perfectly flexible under uniform tension lie in its equilibrium position along the x-axis. The string is pulled aside a short distance and released. If no external forces are present, develop the subsequent motion $u(x, t)$ of the string under the conditions $u(0, t) = u(2, t) = 0, u(x, 0) = sin^3\pi x/2, u_t(x, 0) = 0$.
