LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2014**

MT 3504 - INTEGRAL TRANSFORMS & PARTIAL DIFF. EQUATIONS

Date : 03/11/2014 Dept. No. Max. : 100 Marks

Time : 09:00-12:00

PART – A

ANSWER ALL THE QUESTIONS: **(10 x 2 = 20)**

1. Find the differential equation of all spheres whose centres lie on the z- axis.
2. Solve $\frac{\partial^2 z}{\partial y^2} = \sin y$.
3. Find $L(\sin^2 2t)$
4. Find $L(tSinat)$.
5. Find $L^{-1}\left(\frac{s-3}{(s-3)^2+4}\right)$
6. Find $L^{-1}\left(\frac{s}{s^2+k^2}\right)$
7. If L [f(t)] = F(s) then. Prove that $F\{f(x-a)\} = e^{ias}F(S)$.
8. If L [f(t)] = F(s) then. Prove that $F\{e^{ias}f(x)\} = F(S+a)$.
9. Show that $F_C\{f(ax)\} = \frac{1}{a}F_C\left(\frac{S}{a}\right)$.
10. Prove that $F_S[F_S(x)] = f(S)$.

PART – B

ANSWER ANY FIVE QUESTIONS: **(5 x 8 = 40)**

11. Solve $p(1+q^2) = q(z-1)$.
12. Solve $p^2+q^2 = z^2(x^2+y^2)$
13. Find $L\left(\frac{1-e^t}{t}\right)$
14. Evaluate $\int_0^\infty e^{-2t}\sin 3t dt$
15. Find $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$

16. Find $L^{-1}\left[\frac{s+2}{\left(s^2+4s+5\right)^2}\right]$

17. Show that $F\left\{x^n f\left(x\right)\right\}=\left(-i\right)^n \frac{d^n}{ds^n} F\left\{f\left(x\right)\right\}$

18. Show that $F_C\left\{\frac{1}{\sqrt{x}}\right\}=F_S\left\{\frac{1}{\sqrt{x}}\right\}=\frac{1}{\sqrt{s}}.$

PART – C

ANSWER ANY TWO QUESTIONS: **(2x 20 = 40)**

19. (a) Solve $p^2+q^2-2px-2qy+1=0.$

(b) Solve $\left(y+z\right)p+\left(z+x\right)q=x+y.$

20. Using Laplace transform, solve the equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t$ given that $y=\frac{dy}{dt}=0$ when t =0.

21. (a) Using Laplace transforms, evaluate $\int_0^\infty \frac{\sin^2 tx}{x^2}dx.$

(b) State and prove complex form of Fourier integral theorem.

22. (a) Find the Fourier Cosine transform for F(x) if $f(x)=\begin{cases}1, when\left|x\right|<1\\ 0, when\left|x\right|>1\end{cases}$. Deduce that

(i) $\int_0^\infty \frac{\sin t}{t}dt=\frac{\pi}{2}.$ (ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}.$

(b) Solve the integral equation $\frac{1}{2}\int_0^\infty f\left(t\right)e^{-\left|x-t\right|}dt=h(x)$, where h(x) is a given function.

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