LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2016

16PMT1MC03 / MT 1817 - ORDINARY DIFFERENTIAL EQUATIONS

Date: 07-11-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Answer all questions. Each question carries 20 marks.

1. (a) Determine whether the given sets of functions are linearly dependent or independent. (i) e^x, e^{-x} (ii) $1, x, x^2, \dots, x^n$, (iii) $sinx, sin2x, sin3x$ on $I = [0,2\pi]$. **(5)**

(OR)

(b) With usual notation, prove that $uL(v) - vL(u) = a_0(t)\frac{d}{dt}W[u,v] + a_1(t)W[u,v]$, where *u*, *v* are twice differentiable functions and a_0, a_1 are continuous on *I*. **(5)**

(c) Find the general solution of equation $x'''(t) - x'(t) = e^t$. **(15)**

(OR)

(d) Derive the various solutions of the second order linear homogenous equation with constant coefficients. **(15)**

2. (a) State and prove Laplace's integral representation. **(5)**

(OR)

(b) Let $P_l(x)$ be the Legendre's polynomial. Prove that $P_l(x) = \frac{1}{2^l\, l!}\frac{d^l}{dx^l}(x^2-1)^l$. **(5)**

(c) Solve by Frobenius method, $x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} - y = 0$. **(15)**

(OR)

(d) Derive the orthogonality properties of the Legendre's polynomial. **(15)**

3. (a) Show that $J_{-n}(x) = (-1)^n J_n(x)$ where *n* is a positive or negative integer. **(5)**

(OR)

(b) Obtain the generating function of Bessel's function. **(5)**

(c) Solve: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ where $n \geq 0$. **(15)**

(OR)

(d) Derive the recurrence relations for Bessel's function. **(15)**

4. (a) Argue that all the eigen values of Strum-Liouville problem are real. **(5)**

(OR)

(b) Find the first three approximation of the initial value problem $x'(t) = 2t(1-x)$, $x(0) = 1$, $t \geq 0$. **(5)**

(c) State and prove Picard's theorem for boundary value problem. **(15)**

(OR)

(d) Prove that $x(t)$ is a solution of the equation $L[x(t)] + f(t) = 0$, $a \leq t \leq b$ if and only if $x(t) = \int_a^b G(t,s)f(s)\,ds$ where *G*(*t*, *s*) is the Green's function. **(15)**

5. (a) Explain asymptotically stable solution by an example. **(5)**

(OR)

(b) Define an autonomous system and state the stability behaviours of the system. **(5)**

(c) State and prove the fundamental theorems on the stability of non-autonomous systems. **(15)**

(OR)

(d) Discuss the stability of linear system $x' = Ax$ using Lyapunov's function. **(15)**

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