LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2016**

16UMT1MC01 – ALGEBRA AND CALCULUS - I

Date: 05-11-2016   Dept. No.   Max. : 100 Marks

Time: 01:00-04:00

PART-A

Answer all questions: (10 x 2 =20)

1. Find the nth derivative of e^{-4x}.

2. Find the sub tangent to the curve $y^2 = 4ax$.

3. How to examine a function $f(x, y)$ is minimum?

4. Write down the use of Lagrange's multipliers' method.

5. What is the radius of curvature of the curve $y = x^3 + 8$ at the point (-2,0)?

6. Write down a pedal equation of a curve.

7. Form a rational cubic equation which shall have for roots $1, 3 - \sqrt{-2}$.

8. Define reciprocal equation .

9. Show that the equation $x^5 - 6x^2 - 4x + 5 = 0$ cannot have more than one negative root, using Descartes' rule.

10. Find the upper limit of the positive roots of the equation $2x^3 - 5x^2 + x + 10 = 0$.

PART-B

Answer any FIVE questions   (5 x 8 = 40)

11. If $y = \sin(m \sin^{-1} x)$, Prove that $(1 - x^2)y_2 - xy_1 + m^2 y = 0$ and

$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

12. Find the angle at which the radius vector cuts the curve $\dfrac{l}{r} = 1 + e \cos \theta$.

13. Show that the maximum value of $x^2 y^2 z^2$ subject to the restriction $x^2 + y^2 + z^2 = a^2$

is $\left(\dfrac{a^2}{3} \right)^3$.

14. Find the radius of curvature of the cardioids $r = a(1 - \cos \theta)$.

15. Find the asymptotes of $y^3 - 6xy^2 + 11x^2 y - 6x^3 + x + y = 0$.

16. Solve the equation $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$ of which one root is $-1 + \sqrt{-1}$.

17. Show that the sum of the eleventh powers of the roots of $x^7 + 5x^4 + 1 = 0$ is zero.

18. If the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, equals the sum of the other two roots, prove that $p^3 + 8r = 4pq$.

PART-C

Answer any **TWO** questions: (2 x 20 = 40)

19. (a) Find the nth differential coefficient of $\cos^5 \theta \sin^7 \theta$. (12+8)

   (b) If $xy = ae^x + be^{-x}$,prove that $x\dfrac{d^2 y}{dx^2} + 2\dfrac{dy}{dx} - xy = 0.$

20. Find the maximum or minimum value of the function $x^3 y^2 (6 - x - y).$

21. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3.$

22.(a) Show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in A.P if
$2p^3 - 9pq + 27 = 0.$

   (b) Solve the equation $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$. (10+10)

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