LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2016**

16UMT1MC02 – ANALYTICAL GEOMETRY OF 2D, TRIG. MATRICES

Date: 07-11-2016
Time: 01:00-04:00

Dept. No.

Max. : 100 Marks

PART – A

Answer all questions: (10 X 2 = 20)

1. If $x = \cos\theta + i\sin\theta$ and $\frac{1}{x} = \cos\theta - i\sin\theta$ then find $x^2 + \frac{1}{x^2}$ and $x^2 - \frac{1}{x^2}$.

2. Write the expansion of $x^2 + \frac{1}{x^2}$ in a series of ascending powers of θ.

3. Show that $\cosh^2 x - \sinh^2 x = 1$.

4. Express $\sin ix$ and $\cos ix$ in terms of $\sinh x$ and $\cosh x$.

5. State Cayley – Hamilton theorem.

6. If $A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$ then find the eigen values of A.

7. Find the pole of the line $x + 2y + 4 = 0$ with respect to the parabola $y^2 = 4x$

8. Prove that the sum of the squares of two conjugate semi – diameters of an ellipse is constant.

9. Write the equation of the standard form of the rectangular hyperbola.

10. Write the polar equation of the conic.

PART – B

Answer any FIVE questions: (5 X 8 = 40)

11. Write down the expansion of $\cos 5\theta$.

12. If $\frac{\sin\theta}{\theta} = \frac{5045}{5046}$, show that $\theta = 1°58'$ approximately.

13. Evaluate $\lim\limits_{x\to 0}\left[\dfrac{\tan 2x - 2\tan x}{x^3}\right]$.

14. If $\sin(A + iB) = x + iy$, prove that

(i) $\dfrac{x^2}{\sin^2 A} - \dfrac{y^2}{\cos^2 A} = 1$

(ii) $\dfrac{x^2}{\cosh^2 B} + \dfrac{y^2}{\sinh^2 B} = 1$.

15. Using Cayley - Hamilton theorem find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$

16. Verify Cayley - Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}.$

17. Find the locus of the mid-points of chords of the parabola which subtend a right angle at the vertex of the parabola.

18. Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0.$

PART – C

Answer any TWO questions: (2 x 20 = 40)

19. (i) Express $\dfrac{\sin 6\theta}{\sin \theta}$ in terms of $\cos \theta$.

    (ii) Expand $\sin^4 \theta \cos^2 \theta$ in a series of cosines of multiples of θ. (10+10)

20. (i) If $\tan(x + iy) = u + iv$ prove that $\dfrac{u}{v} = \dfrac{\sin 2x}{\sinh 2y}$.

    (ii) Find the real part and imaginary part of $\tan^{-1}(x + iy).$ (10+10)

21. Diagonalize the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}.$

22. (i) Trace the curve $\dfrac{12}{r} = 4 + \sqrt{3}\cos\theta + \sin\theta.$

    (ii) Show that the locus of the perpendicular drawn from the pole to the tangent to the circle

    $r = 2a\cos\theta$ is $r = a(1 + \cos\theta).$ (10+10)

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