LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2016

MT 1501 – GRAPHS, DIFF. EQU., MATRICES & FOURIER SERIES

Date: 07-11-2016   Dept. No.   Max. : 100 Marks

Time: 01:00-04:00

SECTION – A

(Answer ALL questions)   **(10 × 2 = 20)**

1. Find the equation of the line passing through (2, 9) and (2,-9).

2. Find the domain and range of $f(x) = \dfrac{-2}{x-5}$.

3. State the principle of least square.

4. Reduce $y = ax^n$ to the linear form where a and n are constants.

5. Solve $y_{x+2} - 8y_{x+1} + 15y_x = 0$.

6. Find the particular integral of $y_{n+2} - 4y_{n+1} + 3y_n = 2^n$.

7. Give an example of a symmetric matrix.

8. Define orthogonal matrix.

9. Find the Fourier constant a_0 for $f(x) = x^3$ in $-\pi < x < \pi$.

10. Define half range Fourier series.

SECTION – B

(Answer any FIVE questions)   **(5 × 8 = 40)**

11. The demand for a certain commodity is related to the price by $p = 80 - (\tfrac{2}{3})x$. The supply is related to the price by $p = \dfrac{4}{3}x$. Find the equilibrium demand and the equilibrium price.

12. (a) Complete the square and graph the function $f(x) = -x^2 + 6x - 12$.

    (b) The price and the demand for an item are related by $p = 32 - x^2$, while price and supply are related by $p = x^2$. Find the equilibrium supply.

13. Use the method of least squares to fit a straight line to the following data:

    x :   0   5   10   15   20

    y :   7   11   16   20   26

    Estimate the value of y when x = 25.

14. Find the difference equation satisfied by $y = ax^2 - bx$.

15. Solve $y_{x+2} - 6y_{x+1} + 8y_x = 4^x$.

16. Verify Caley Hamilton theorem for the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$.

17. Find all eigen vectors of the matrix $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$.

18. Obtain Fourier expansion for the function $f(x) = \tfrac{1}{2}(\pi - x)$, $0 < x < 2\pi$.

1

(Answer any TWO questions) $\qquad$ **(2 × 20 =40)**

19. (a) When a company sells x units of a product, its profit is $P(x) = -2x^2 + 40x + 280$. Find (i) the number of units that should be sold so that maximum profit is received and (ii) the maximum profit.

  (b) Convert the equation $y = \dfrac{x}{a+bx}$ to a linear form and hence determine the values of a and b which will best fit the following data:

  x : 8   10   15   20   30   40

  y : 13   14   15.4   16.3   17.2   17.8 .

20. (a) Solve the difference equation $u(x+2) - 4u(x) = 9x^2$.

  (b) Solve $y_{n+2} + y_{n+1} - 56\, y_n = 2^n (n^2 - 3)$.

21. Diagonalize the matrix $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.

22. (a) Obtain a Fourier expansion for $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi. \end{cases}$

  (b) Find the Fourier series for $f(x) = |x|$ in $-\pi < x < \pi$ and deduce that $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \cdots = \dfrac{\pi^2}{8}$.

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