LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2016

MT 1816 - REAL ANALYSIS

Date: 04-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all Questions. All questions carry equal marks.

1. (a) State and prove the fundamental theorem of calculus.

(OR)

(b) If P^* is a refinement of P then prove that $L(P,f,\propto) \leq L(P^*,f,\propto)$ and $U(P^*,f,\propto) \leq U(P,f,\propto)$. **(5 marks)**

(c) State and prove a necessary condition and sufficient condition for a bounded real valued function to be a Riemann-Steiltjes integrable.

(OR)

(d) Assume α increases monotonically and $\alpha' \in$ □ on [a, b]. Let f be a bounded real function on [a,b]. Then prove that $f \in \mathfrak{B}(\alpha)$ if and only if $f\alpha'$ ∈ □. Also prove that $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$. **(15 marks)**

2. (a) State and prove the Cauchy criterion for uniform convergence of sequence of functions.

(OR)

(b) Prove that for $f_n(x) = n^2x(1-x^2)^n, 0 \leq x \leq 1, n = 1,2 \ldots$,

$$\int_0^1 (\lim_{n\to\infty} f_n(x))\, dx \neq \lim_{n\to\infty} \int_0^1 f_n(x)dx.$$ **(5 marks)**

(c) State and prove the Stone-Weierstrass theorem.

(OR)

(d) If $\{f_n\}$ is a sequence of differentiable functions on [a, b] such that $\{f_n(x_0)\}$ converges for $x_0 \in [a,b]$ and $\{f_n'\}$ converges uniformly on [a, b] then prove that $\{f_n\}$ converges uniformly on [a, b] to a function f and $\lim_{n\to\infty} f_n'(x) = f'(x)$. **(15 marks)**

3. (a) Let $S = \{\varphi_0, \varphi_1, \varphi_2, ..\}$, where $\varphi_0(x) = \frac{1}{\sqrt{2\pi}}, \varphi_{2n-1}(x) = \frac{\cos nx}{\sqrt{\pi}}$ and $\varphi_{2n}(x) = \frac{\sin nx}{\sqrt{\pi}}$, for n = 1, 2…. Prove that S is orthnormal on any interval of length 2 π.

(OR)

(b) State and prove the Bessel's Inequality and Parseval's formula. **(5 marks)**

(c) (i) Define Dirichlet's kernel and prove that $\frac{1}{2} + \sum_{k=1}^{n} coskx = \frac{\sin(2n+1)\frac{x}{2}}{2sin\frac{x}{2}}, x \neq 2m\pi$

(ii) If $f \in L[0,2\pi]$, f is periodic with period 2π and $\{s_n\}$ is a sequence of partial sums of Fourier series generated by f, $s_n = \frac{a_0}{2} + \sum_{k=1}^{n}(a_k coskx + b_k sinkx), n = 1,2 \ldots$ then prove that

$$s_n(x) = \frac{2}{\pi}\int_0^{\pi} \frac{f(x+t)+f(x-t)}{2} D_n(t)dt.$$ **(5+10 marks)**

(OR)

(d) State and prove the Riemann-Lebesgue lemma and use the lemma to prove the following:

for f□ $L(-\infty,+\infty)$, $\lim_{□\to\infty} \int_{-\infty}^{\infty} f(t)\frac{1-cos\propto t}{t}dt = \int_0^{\infty} \frac{f(t)-f(-t)}{t}dt$. (15 marks)

4. (a) If A, B $\in$ L(R^n, R^m) and c is a scalar, then prove that, $\|A + B\| \leq \|A\| + \|B\|$ and $\|cA\| = |c|\|A\|$

(OR)

(b) Suppose X is a complete metric space and ϕ is a contraction of X into X. Prove that there exist one and only one $x \in X$ such that $\phi(x) = x$.. **(5 marks)**

(c) State and prove the inverse function theorem.

(OR)

(d) State and prove the implicit function theorem. **(15 marks)**

5. (a) Define heat flow and the heat equation.

(OR)

(b) Explain rectilinear coordinate system with algebraic and geometric approach.

(5 marks)

(c) Derive the expression for Newton's Law of Cooling.

(OR)

(d) Derive the D' Alembert's wave equation for a vibrating string. **(15 marks)**

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