LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2016

MT 1818 - DIFFERENTIAL GEOMETRY

Date: 10-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer ALL the Questions:

1. a) Derive the equation of the osculating plane. **(5)**

OR

b) Prove that the curvature is the rate of change of angle of contingency with respect to the arc length. **(5)**

c) Find the unit vectors $\vec{t}, \vec{n}, \vec{b}$ of the curve $\vec{x} = (u, u^2, u^3)$ at the point $u = 1$. Find also the equation of the tangent, principal normal and binormal at this point. **(15)**

OR

d) State and prove Serret-Frenet formula and express $\bar{t}, \bar{n}$ and $\bar{b}$ in terms of Darboux vector. **(15)**

2. a) Find the necessary and sufficient condition that a curve to be a helix. **(5)**

OR

b) Find the plane that has three point contact at the origin with the curve $x = u^4 - 1, y = u^3 - 1, z = u^2 - 1$. **(5)**

c) Derive the equation of involute of a curve. Also find the curvature and torsion of an involute. **(15)**

OR

d) Find the curve whose intrinsic equations are $\kappa = \frac{1}{\sqrt{2as}}$ and $\tau = 0$. **(15)**

3. a) Define the envelope of the system of surface. Find the envelope of the sphere $(x - acos\theta)^2 + (y - asin\theta)^2 + z^2 = b^2$. **(5)**

OR

b) Prove that the first fundamental form of a surface is positive definite. **(5)**

c) Derive any two developable associated with space curves. **(15)**

OR

d) Prove that the necessary and sufficient condition for the surface may be developable is that its Gaussian curvature is zero. **(15)**

4. a) State and prove Meusnier's theorem. **(5)**

OR

b) Find the principal radii of curvature of the surface $ycos\left(\frac{z}{a}\right) = xsin\left(\frac{z}{a}\right)$. **(5)**

c) (i) State and prove Euler's theorem.

(ii) Prove that the curves of the family $\frac{v^3}{u^2} = a\ constant$ are geodesic on a surface with metric $v^2du^2 - 2uvdudv + 2u^2dv^2$ $(u > 0, v > 0)$. **(7 + 8)**

OR

d) Define Dupin Indicatrix. Derive equation of Dupin Indicatrix. **(15)**

5. a) Prove that the sphere is the only surface in which all points are umbilics. **(5)**

OR

b) Derive Weingarten's equations. **(5)**

c) Derive the intrinsic formula for Gaussian curvature. **(15)**

OR

d) Derive partial differential equation of surface theory. **(15)**

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