LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **NOVEMBER 2016**

MT 2502 – ALGEBRA AND CALCULUS - II

Date: 12-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

PART-A

Answer ALL the questions (10 x 2=20)

1. Evaluate $\int \sec^6 x\,dx$.
2. Evaluate $\int_0^1 x^3\left(1-x^2\right)^{-\frac{1}{2}}dx$.
3. Evaluate $\int_1^2\int_1^x xy^2\,dy\,dx$.
4. If $u = x^2 - y^2$ and $v = x^2 + y^2$, then find $\frac{\partial(x,y)}{\partial(u,v)}$.
5. Prove that $\beta(m,n) = \beta(n,m)$.
6. Evaluate $\int_0^{\frac{\pi}{2}} \sin^7\theta\cos^7\theta\,d\theta$ by using gamma function.
7. Test for convergency the series $\sum_{n=0}^{\infty}\frac{n^3+1}{2^n+1}$.
8. State Raabe's Test.
9. Expand $(1-x)^{-3}$ and $(1+x)^{\frac{1}{2}}$.
10. Expand $\frac{e^x+e^{-x}}{2}$ and $\log(1+x)$.

PART-B

Answer any FIVE questions (5 x 8=40)

11. Evaluate $\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$.
12. Find the perimeter of the cardioid $r = a(1+\cos\theta)$.
13. Evaluate $\iint_R (x-y)^4 e^{x+y}\,dx\,dy$ where R is the square with vertices $(1,0),(2,1),(1,2)$ and $(0,1)$.
14. Prove that $\int_0^1 x^m\left(1-x^n\right)^p dx$ in terms of gamma function and evaluate $\int_0^1 x^{\frac{3}{2}}(1-x)^{\frac{5}{2}}dx$.

15. Test for the convergency or divergency the series $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$.

16. Examine the convergence of $\frac{1^2}{2^2}+\frac{1^2}{2^2}.\frac{3^2}{4^2}+\frac{1^2}{2^2}.\frac{3^2}{4^2}.\frac{5^2}{6^2}+....$.

17. Prove that $\sum_{n=0}^{\infty}\frac{5n+1}{(2n+1)!}=\frac{e}{2}+\frac{2}{e}$.

18. Show that $\frac{1}{1.2.3}+\frac{5}{3.4.5}+\frac{9}{5.6.7}+.......=\frac{5}{2}-3\log 2$.

PART-C

Answer any TWO questions (2 x 20=40)

19. (a) If $\int_0^{\frac{\pi}{2}}\cos^m x\cos nx\,dx=f(m,n)$, prove that $f(m,n)=\frac{m}{m+n}f(m-1,n-1)$. also prove that $f(n,n)=\frac{\pi}{2^{n+1}}$ (10)

(b) Evaluate $\int(\log x)^3x^2dx$. (10)

20. (a) Evaluate $\iiint xyz\,dx\,dy\,dz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$ by transforming into spherical coordinates. (10)

(b) Use the substitution $x+y+z=u, y+z=uv, z=uvw$ to evaluate the integral $\iiint\left[xyz(1-x-y-z)\right]^{1/2}dx\,dy\,dz$ taken over the tetrahedral volume enclosed by the planes $x=0, y=0, z=0$ and $x+y+z=1$ (10)

21. (a) State and prove the relation between gamma and beta functions. (10)

(b) Prove that $\int_0^{\infty}\frac{t^2}{1+t^4}dt=\frac{\pi}{2\sqrt{2}}$. (10)

22. (a) Discuss the convergence of the series $\frac{1}{1+x}+\frac{1}{1+2x^2}+\frac{1}{1+3x^3}+...$ for positive values of x. (10)

(b) Find the sum to infinity of the series $\frac{3.5}{1!}x+\frac{4.6}{2!}x^2+\frac{5.7}{3!}x^3+...\infty$ (10)

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