LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **NOVEMBER 2016**

MT 2503 – ANALY. GEOM. OF 3D, FOURIER SERIES & NUM. THEORY

Date: 14-11-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

Answer ALL questions. **(10 × 2 = 20)**

1. Find the angle between the planes $2x - y + z = 6$, $x + y + 2z = 3$.
2. State the equation of the straight line joining two points (x_1,y_1,z_1) and (x_2,y_2,z_2).
3. Find the equation of the sphere with centre (-1,2,-3) and radius 3 units.
4. What is the general equation of the sphere passing through a circle?
5. Find the Fourier coefficient a_0 for the function $f(x) = x^2$ in the interval $(-\pi, \pi)$.
6. Give the Fourier series expansion of an odd function.
7. Find the number of divisors of 480 excluding 1 and 480.
8. State Wilson's theorem.
9. State Cauchy's inequality.
10. Show that $n^n > 1 . 3 . 5 . \; (2n - 1)$.

PART – B

Answer any FIVE questions **(5 × 8 = 40)**

11. Find the equation of the plane which passes through the point (-1,3,2) and perpendicular to the planes $x+2y+2z=5$ and $3x+3y+2z=8$.
12. Find the symmetric form of the equation of the straight line which is the intersection of the planes $x+5y-z=7$ and $2x-5y+3z+1=0$.
13. Find the equation of the sphere having the circle $x^2+y^2+z^2-2x+4y-6z+7=0$, $2x-y+2z=5$ for a great circle.
14. Find the equation of the sphere which touches the sphere $x^2+y^2+z^2-6x+2z+1=0$ at the point (2,-2,1) and passes through the origin.
15. Express $f(x) = \frac{1}{2}(\pi-x)$ as a Fourier series with period 2π in the interval $[0,2\pi]$.
16. If $d_1, d_2, \ldots d_r$ (including 1 and N) are the divisors of N , then show that $\varphi(d_1)+\varphi(d_2)+\ldots\varphi(d_r)=N$.
17. Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.

18. Prove that $8xyz < (x+y)(y+z)(z+x) < \frac{8}{3}(x^3 + y^3 + z^3)$.

PART – C

Answer any TWO questions. **(2 × 20 =40)**

19. (a) Show that the origin lies in the acute angle between the planes $x+2y+2z = 9$, $4x - 3y + 12z + 13=0$. Find the planes bisecting the angles between them and point out which bisects the obtuse angle.

(b) Find the shortest distance between the lines

$$\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}, \quad \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}.$$

20. (a) Find the equation of the sphere passing through the points (2,3,1), (5,-1,2), (4,3,-1) and (2,5,3).

(b) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2 -2x-4y=0; x+2y+3z=8$ and touches the plane $4x+3y=25$.

21.(a) If $f(x) = -x$ in $-\pi < x < 0$ $= x$ in $0 \leq x < \pi$, expand f(x) as Fourier series in the interval $(-\pi, \pi)$ and deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$.

(b) If x and y are primes to the prime number n, show that $x^{n-1} - y^{n-1}$ is divisible by n. Deduce that $x^{12} - y^{12}$ is divisible by 1365.

22. (a) If $M = 1 . 3. 5 . . . (p-2)$ where p is an odd prime, show that $M^2 \equiv (-1)^{\frac{p+1}{2}} \pmod{p}$.

(b) If x and y are positive quantities whose sum is 4, show that $(x + \frac{1}{x})^2 + (y + \frac{1}{y})^2 \geq 12\frac{1}{2}$.

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