LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - MATHEMATICS

SECOND SEMESTER - NOVEMBER 2016

Date: $11-11-2016$ Dept. No.	MT 2811 - MEASURE THEORY AND INTEGRATION						
 (a) Let f and g be measurable functions on the set E then prove that f + g and fg are measurable functions. (DR) (b) Prove that the class M of Lebesgue measurable sets is a Sigma Algebra. (c) Prove that the outer measure of an interval equals to its length. (d) (i) Prove that not every measurable set is a Borel set. (ii) Prove that every interval is measurable. (7+8) (a) Show that lim ∫₀[∞] dx / (1+x/2)^{nx+1/n} = 1. (b) Show that ∫₀[∞] sint / (1+x/2)^{nx+1/n} = 1. (c) If f and g be integrable functions, then prove the following: (i) af is integrable and af dx = a∫ f dx. (ii) f + g is integrable and f(f + g) dx = ∫ f dx + ∫ g dx (iii) If f = 0 a.e., then f dx = 0. (iv) If A and B are disjoint measurable sets, then ∫_A f dx + ∫_B f dx = ∫_{AUB} f dx (d) Let f be a bounded function defined on the finite interval [a, b], then prove that f is Riemann integrable over [a, b] if and only if it is continuous a.e. (15) (a) Show that every algebra is a ring and every σ-algebra is a σ-ring. (b) Define measure, outer measure and complete measure on a ring R and show that if A, B ∈ R and A ⊆ B then μ(A) ≤ μ(B). (c) If μ is a σ-inite measure on a ring , then prove that it has a unique extension to the σ-ring S(R). (d) Define a complete measure. Let μ* be an outer measure on R(R) and let S* denote the class of μ* - measurable sets. Prove that S* is a σ-ring and μ* restricted to S* is a 			Dept. No.	Max.: 100 Marks			
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4. (a) Define a convex function and prove that for a convex function ψ on (a,b) such that

	$a < s < t < u < b$, the $\psi(s,t) \leq \psi(s,u)$.	(5)				
	(OR)					
	(b) State and prove Jensen's Inequality.	(5)				
	(c) (i) State and prove Holder's Inequality. (ii) Let ψ be a function on (a,b) . Then prove that ψ is convex on (a,b) if and only that $a < x < y < b$, the graph of ψ on (a,x) and (y,b) does not lie below the lin $(x,\psi(x))$ and $(y,\psi(y))$.					
	(OR)					
	(d) State and Prove Minkowski's inequality.	(15)				
5.	. (a) Prove that the countable union of sets with respect to a signed measure v is a posit	tive set.				
	(OR)					
	(b) Let v be a signed measure on $[X, S]$. Then prove that there exists a positive and a negative set B such that $A \cup B = X$, $A \cap B = \Phi$.	set <i>A</i> (5)				
	(c) State and prove Radon-Nikodym Theorem.	(15)				
	(OR)					
		15)				