LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2016**

MT 3503 – VECTOR ANALYSIS & ORDINARY DIFF. EQUATIONS

Date: 04-11-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART-A

Answer all questions: (10 x 2 =20)

1. Prove that $Div(\vec{r}) = 3,$ where $\vec{r}$ is the position vector.

2.If $F = xy\vec{i} + yz\vec{j} + zx\vec{k},$ show that $\nabla^2 F = 0.$

3. If $F = y\vec{i} - x\vec{j},$ evaluate $\int_C \vec{F}.d\vec{r}$ from (0,0) to (1,1) along the curve $y = x.$

4. Define volume integral.
5. State Green's theorem.
6. State Stoke's theorem.
7. Define differential equation.
8. Write down the Bernoulli's equation.
9. Solve $(D^2 - 3D + 2)y = 0.$
10. Define Legendre linear equation.

PART-B

Answer any FIVE questions (5 x 8 = 40)

11. Compute the divergence and curl of the vector $\vec{F} = xy^2\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$

at (1,-1,1).

12. Prove that $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla.\vec{F}) - \nabla^2\vec{F}.$

13. Evaluate $\iiint_V \nabla.F \, dv$, where $F = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and V is the volume enclosed by the cube $0 \le x, y, z \le 1.$

14. Evaluate $\iint_S \vec{F}.\hat{n}\,ds$, where $F = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and S is that part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant.

15. Evaluate $\int_C \vec{F}.d\vec{r}$, for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle bounded by $x = \pm a, y = 0, y = b.$ Using Stokes' theorem.

16. Solve $xp^2 - 2yp + x = 0.$

17. Solve $(D^2 + 4D + 4)\,y = e^{-2x}$.

18. Solve $(D^2 + 4)\,y = \sin x$.

PART-C

Answer any **TWO** questions: (2 x 20 = 40)

19. (a) Prove that $F = (y^2 \cos x + z^3)\vec{i} + (2y \sin x - 4)\vec{j} + (3xz^2 + 2)\vec{k}$ is irrotational and find its scalar potential. (12+8)

(b) Evaluate $\int_C 2xyz^2 d\vec{r}$ where c is the curve $x = t^2, y = 2t, z = t^3$ from t=0 to t=1.

20. Verify Gauss Divergence theorem for the function $\overline{F} = 2xz\vec{i} + yz\vec{j} + z^2\vec{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$.

21. Solve $(1 - x^2)\frac{dy}{dx} + 2xy = x\sqrt{1 - x^2}$, given that y=0 when x=0.

22. Solve $x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = \log x + \cos(\log x).$

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