LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – NOVEMBER 2016

MT 3810 - TOPOLOGY

Date: 01-11-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer all the questions

I. a) 1) In any metric space prove that each open sphere is an open set. (3)

OR

a) 2) In any metric space prove that each closed sphere is a closed set. (3)

b) 1) State and prove Cantor's intersection theorem.

b) 2) Derive Cauchy and Minkowski's inequalities. (7+10)

OR

c) 1) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous $\Leftrightarrow f^{-1}(G)$ is open whenever G is open in Y. What is special about this result in the study of topological spaces.

c) 2) Let X be a metric space and let Y be a complete metric space, and let A be a dense subspace of X. If f is uniformly continuous mapping of A into Y then prove that f can be extended uniquely to a uniformly continuous mapping g of X into Y. (7+10)

II. a) 1) Prove that every separable metric space is second countable. (3)

OR

a) 2) Prove that any continuous image of a compact space is compact. (3)

b) 1) State and prove Lindelof's theorem.

b) 2) State and prove Heine Borel theorem. (7+10)

OR

c) 1) Prove that a topological space is compacat iff every class of closed sets with the finite intersection property has non-empty intersection.

c) 2) Prove that a topological space is compact if every subbasic open cover has a finite subcover (4+13)

III. a) 1) Prove that the product of any non-empty class of Hausdorff space is a Hausdorff space. (3)

OR

a) 2) Prove that a one-to-one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism. (3)

b) Prove that the following statements are equivalent:(Lebesgue's Covering Lemma need not be proved)

(i) X is compact (ii) X is sequentially compact and (iii) X has the Bolzano-Weierstrass property. (17)

OR

c) 1) Quoting the necessary results prove that a metric space is compact iff it is complete and totally bounded.

c) 2) State and prove Ascoli's theorem. (5+12)

IV. a) 1) Prove that any continuous image of a connected space is connected (3)

OR

a) 2) Prove that a topological space X is disconnected iff there exists a continuous mapping of X onto the discrete two- point space $\{0,1\}$. (3)

b) 1) Proving the necessary results prove the theorem: Let X be a normal space and let A and B be disjoint closed subspace of X. If [a,b] is any closed interval on the real line, then prove that there exists a continuous real function f defined on X, all of whose values lie in [a.b] such that f(A) = a and f(B) = b.

b) 2) Prove that the product of any non-empty class of connected space is connected. (12+5)

OR

c) State and prove Urysohn Imbedding theorem. (17)

V. a) 1) Prove that X_∞ is compact. (3)

OR

a) 2) Prove that X_∞ is Hausdorff. (3)

b) State and prove Weierstrass approximation theorem. (17)

OR

c) Proving the necessary lemmas, state and prove Real Stone-Weierstrass theorem. (17)

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