LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2016**

MT 3965 – NUMERICAL ANALYSIS

Date: 09-11-2016   Dept. No. ☐   Max. : 100 Marks
Time: 09:00-12:00

Answer ALL the questions.

1. a) Discuss the rate of convergence of Newton Raphson Method.

    OR

    b) Find a real root of the equation $x^3 - 2x - 5 = 0$ using secant method correct to two decimal places.
    (5)

    c) Find a root of the equation $x^3 - x - 11 = 0$ correct to three decimal places using bisection method.
    (15)

    OR

    d) Find a real root of the equation $xe^x = 3$ by the Regula falsi method correct to three decimal places.
    (15)

2. a) Find the cubic polynomial which takes the following values: $y(1) = 24$, $y(3) = 120$, $y(5) = 336$ and $y(7) = 720$.
    (5)

    OR

    b) Find the third order Hermite polynomial passing through the points (x_i, y_i, m_i), $i = 0,1$.
    (5)

    c) Derive Gauss forward formula for central differences.
    (15)

    OR

    d) The following table gives the values of e^x for certain equidistant values of x. Find the value of e^x when $x = 0.644$ and $x = 0.638$ using Stirling's and Bessel's formulae.

x	$y = e^x$
0.61	1.840431
0.62	1.858928
0.63	1.877610
0.64	1.896481
0.65	1.915541
0.66	1.934792
0.67	1.954237

3. a) From the following table, find x, correct to two decimal places, for which y is maximum and find this value of y.

x	y
1.2	0.9320
1.3	0.9636
1.4	0.9855
1.5	0.9975
1.6	0.9996

(5)

OR

b) Discuss about the total error of the trapezoidal formula and derive it. (5)

c) Calculate the first and second derivative of the function at $x = 1.6$ from the following table and also estimate the errors in the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.6$.

x	y
1.0	2.7183
1.2	3.3201
1.4	4.0552
1.6	4.9530
1.8	6.0496
2.0	7.3891
2.2	9.0250

(15)

OR

d) Evaluate $\int_{0}^{-10} \frac{dx}{1+x^2}$ by using (i) Trapezoidal rule (ii) Simpson's 1/3 rule and (iii) Simpson's 3/8 rule with $h = 1$. (15)

4. a) Solve the system of equations $2x + y + z = 10; 3x + 2y + 3z = 18$ and $x + 4y + 9z = 16$ using Gauss elimination method.

OR

b) Compute the inverse of the matrix $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$. (5)

c) Solve the equations $2x + 3y + z = 9; x + 2y + 3z = 6$ and $3x + y + 2z = 8$ by LU decomposition. (15)

OR

d) Solve by Jacobi iteration method the system of equations $8x - 3y + 2z = 20; 4x + 11y - z = 33$ and $6x + 3y + 12z = 35$. (15)

5. a) Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ with the initial condition $y = 0$ when $x = 0$, use Picard's method to obtain y for $x = 0.25$ correct to three decimal places. (5)

OR

b) From the Taylor series for $y(x)$, find $y(0.1)$ correct to three decimal places if $y(x)$ satisfies $y' = x - y^2$ and $y(0) = 1$. (5)

c) Given $\frac{dy}{dx} = 1 + y^2$, where $y = 0$ when $x = 0$, find $y(0.2), y(0.4)$ and $y(0.6)$ using Runge-Kutta method of fourth-order. (15)

OR

d) Solve $\frac{dy}{dx} = 1 - y, y(0) = 0$ in the range $0 \le x \le 0.3$ using (i) Euler's method and (ii) Modified Euler's method by choosing $h = 0.1$. (15)

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