LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

FOURTH SEMESTER – NOVEMBER 2016

MT 4200 - ADVANCED MATHEMATICS FOR PHYSICS

Date: 11-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

PART – A

Answer ALL questions. (10 × 2 = 20)

1. State Bernoulli's formula.
2. Write any two properties of definite integral.
3. Solve $(D^2 + 4D + 4)y = 0$.
4. Define exact differential equation.
5. State the relation between Beta and Gamma integral.
6. If $u = (x - y)(y - z)(z - x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
7. Find a unit vector normal to the surface $x^2 + y^2 - z = 10$ at (1,1,1).
8. State Greens theorem.
9. Define a cyclic group.
10. Define Kronecker's delta.

PART – B

Answer any FIVE questions. (5 × 8 = 40)

11. Solve $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}}}{(\sin x)^{\frac{3}{2}} + (\cos x)^{\frac{3}{2}}} dx = \frac{\pi}{4}$.
12. Evaluate $\int x^4 \sin x \, dx$.
13. Solve $\frac{dy}{dx} + y\cos x = \frac{1}{2}\sin 2x$.
14. Solve $(D^2 + 16)y = \cos 4x$.
15. Change the order of integration and hence evaluate $\int_1^3 \int_{y=0}^{\frac{6}{x}} x^2 \, dy dx$.
16. Find $div\ curl\ \vec{F}$ if $\vec{F} = x^2 y\vec{i} + xz\vec{j} + 2yz\vec{k}$.
17. Show that the union of two subgroups of G is a subgroup iff one is contained in other.
18. Evaluate $\int \sqrt{2x^2 - 7x + 5}\, dx$.

PART – C

Answer any TWO questions. (2 × 20 = 40)

19. (a) Find the Fourier series to represent $x - \pi$ in the interval $(-\pi, \pi)$.
    (b) Find a sine series for $f(x) = c$ in the range 0 to π. (15+5)
20. Solve $(D^2 + 4D + 5)y = e^x + x^3 + \cos 2x$. (20)
21. (a) By transforming into polar coordinates, evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dxdy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ $(b > a)$.

(b) Solve $\int_0^{\frac{\pi}{2}} \sin \theta \cos \theta \, d\theta$. (15+5)

22. (a) Verify Gauss divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

(b) If A_i and B_j are covariant vectors. Show that A_iB_j is a covariant tensor of order 2.

(15+5)