LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

FOURTH SEMESTER – NOVEMBER 2016

MT 4203 - ADVANCED MATHEMATICS FOR PHYSICS

Date: 11-11-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Part A

*Answer **ALL** the questions* *(10 x 2 =20)*

1. Evaluate $\int \frac{dx}{a^2-x^2}$.
2. Define Fourier series.
3. Solve $(D^2 + 5D + 6)\, y = 0$.
4. Write down the transformation from Cartesian to polar co-ordinates.
5. Find the particular integral of $(3D^2 + D - 14)y = 13e^{2x}$
6. State the relation between Beta and Gamma function.
7. Prove that the vector $\bar{f} = (x + 3y)\bar{\imath} + (y - 3z)\,\bar{\jmath} + (x - 2z)\bar{k}$ is solenoidal.
8. State Stokes theorem.
9. Define group.
10. Define Kronecker's delta.

Part B

*Answer any **FIVE** questions* *(5 x 8 = 40)*

11. *Evaluate* $\int x^3 \cos 2x\, dx$.
12. Find a sine series for $f(x) = x$ in the range 0 to π.
13. Solve $(D^2 + D + 1)\, y = x^2$.
14. *Solve* $\frac{dy}{dx} + y \cos x = \frac{1}{2}\sin 2x$.
15. Evaluate $\iint_R x\, y\, dx\, dy$, where R is the region in the first quadrant bounded by the hyperbolas $x^2 - y^2 = a^2$ and $x^2 - y^2 = b^2$ and the circles $x^2 + y^2 = c^2$ and $x^2 + y^2 = d^2$ $(0 < a < b < c < d)$.
16. *Solve* $dy - y\, dx = \sqrt{x^2 + y^2}\, dx$.
17. *If* $\bar{F} = xy^2\bar{\imath} + 2x^2yz\bar{\jmath} - 3yz\bar{k}$, *find div* $\bar{F}$ *and curl* $\bar{F}$ *at* $(1, -1, 1)$.
18. *Prove that* $\{1, -1, i, -i\}$ *is an abelian multiplicative finite group of order 4.*

Part C

Answer Any **TWO** Questions. (2 x 20 = 40)

19. (a) Find the Fourier series to the function $f(x) = \frac{1}{2}(\pi - x)$ in the interval $(0, 2\pi)$.

    (b) Derive the relationship between Beta and Gamma functions. (12 + 8)

20. Solve $(D^2 + 4D + 5)y = e^x + x^3 + \cos 2x$.

21. (a) Evaluate $\iint xy\, dx\, dy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$.

(b) Change the order of integration and evaluate $\int_{0}^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$. (10 + 10)

22.(a) Verify Green's theorem for $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region x = 0, y = 0, x + y = 1.

(b) Define cyclic group and prove that every cyclic group is abelian. (12 +8)