LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **NOVEMBER 2016**

MT 4502/MT 4500 – MODERN ALGEBRA

Date: 04-11-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

ANSWER ALL QUESTIONS (10 x 2 = 20)

1. Define one-to-one correspondence between two sets A and B. Give an example.
2. State the Fundamental Theorem of Arithmetic.
3. Define order of an element of a group.
4. Define cyclic group and give an example.
5. Define automorphism of a group with an example.
6. Define odd and even permutation.
7. Define a division ring.
8. What is a field?
9. Define maximal ideal.
10. State Unique factorization theorem.

PART – B

ANSWER ANY FIVE QUESTIONS. (5 x 8 = 40)

11. Show that the set Q^+ of all positive rational numbers form a group under the operation * defined by $a*b = ab/2$ for all a, b in Q^+.
12. If H and K are subgroups of a group G, then prove that HK is a subgroup of G if and only if $HK = KH$.
13. Let G be a cyclic group of order n with generator a, then show that a^m is also a generator of G if and only if m and n are relatively prime.
14. Show that any infinite cyclic group G is isomorphic to the group Z of integers under addition.
15. Let G be a group. Prove that $I(G)$, the set of all inner automorphisms of G, is a normal subgroup of $A(G)$, the group of all automorphisms of G.
16. Show that the set of all 2 X 2 matrices over integers is an infinite noncommutative ring with unity.
17. Prove that the intersection of two subfields of a field F is a subfield of F.
18. Prove that every field is a PID.

PART – C

ANSWER ANY TWO QUESTIONS. (2 x 20 = 40)

19. a. Find the group of all symmetries of an equilateral triangle.

    b. If H and K are finite subgroups of a group G, then prove that $o(HK) = \dfrac{o(H)o(K)}{o(H \cap K)}$.
20. a. State and prove Lagrange's theorem.

    b. State and prove Fundamental Homomorphism Theorem for groups.
21. a. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.

    b. If p is prime then prove that Z_p is a field.
22. a. Prove that the characteristic of an integral domain D is either zero or a prime number.

    b. State and prove the Unique Factorization theorem on a Euclidean ring.

****