LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **NOVEMBER 2016**

MT 4503 – ALGEBRAIC STURUCTURE - I

Date: 04-11-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

Answer **ALL** the questions: (10 x 2 = 20 marks)

1. Define an equivalence relation and give an example.
2. Prove that in a group G, the identify element is unique.
3. Show that every cyclic group is abelian.
4. Define a normal subgroups of a group.
5. State the fundamental homomorphism theorem.
6. It G is a group of order 2, prove that the only automorphism of G is the identify map.
7. Define an integral domain.
8. Give an example of a subring in a ring R, Which is not an ideal of R.
9. Give an example of an Euclidean ring.
10. What are Gaussian integers?

PART – B

Answer any **FIVE** questions: (5 x 8 = 40 marks)

11. Prove that if H and K are subgroups's of G, then HK is a subgroup of G if and only if HK = KH.
12. Show that every subgroup of a cyclic group is cyclic.
13. Show that every group of prime order is cyclic.
14. State and prove Lagrange's theorem.
15. Prove that any permutation of a finite set can be expressed as a product of transpositions.
16. Show that every finite integral domain is a field.
17. Show that every Euclidean ring is a principal ideal domain.
18. Find a greatest common division of a = 14 – 3i and b = 4+7i and represent in the form $\lambda a + \mu b$ in Z(i).

PART – C

Answer any **TWO** questions: (2 x 20 = 40 marks)

19. a) Let H and K are subgroups of G, Show that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$.

    b) Show that union of two subgroups of a group G is a subgroup of G if and only if one is contained in the other. (12+8)

20. a) Show that every subgroups of an abelian subsgroup is normal.
    b) Is the intersection of two normal subgroups of G a normal subgroup of G? Justify.
    c) State and prove the fundamental theorem of group homomorphism. (4+4+12)

21. a) Prove that every group is isomorphic to a group of permutations.
    b) If R is a commutative ring with unit element whose only ideals are (0) and R itself, show that R is a field. (10 + 10)

22. a) Let R be a commutative ring work unity and P an ideal of R. Then prove that P is a prime ideal of R of and only if R/P is an integral domain.

    b) If a = 14 – 3i and b = 4 + 7i, find Gaussian integers q and r such that a = q b+r, where r = 0 or d(r) < d (b). (12 + 8)

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