LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2016**

MT 5508/MT 5502 – LINEAR ALGEBRA

Date: 17-11-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL questions: (10 x 2 = 20 marks)

1. Show that the vectors (0,1,1), (0,2,2) and (1,5,3) in R^3 are linearly independentent over R.
2. Is the union of subspaces a subspace? Justify.
3. Define rank and nullity of a vector space homomorphism T : U $\rightarrow$ V, where U and V are vector spaces over a field F.
4. For a homomorphism T: V $\rightarrow$ V, prove that Kernal of T is a subspace of V.
5. Define an inner product space.
6. Let T $\in$A(V) and $\lambda\in$ F. If λ is an eigen value of T, prove that λ I – T is singular.
7. Show that the matrix $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.
8. If A and B are Hermitian, then show that AB – BA is skew Hermitian.
9. If T $\in$ A (V) is Hermitian, then show that all its eigen values are real.
10. Find the rank of the matrix $A = \begin{pmatrix} 1 & 5 & -7 \\ 3 & 8 & 5 \end{pmatrix}$ over the field of rational numbers.

PART – B

Answer any FIVE questions: (5 x 8 = 40 marks)

11. Prove that the intersection of two sub-spaces of a vector space V is a subspace of V.
12. Give a characterization of a nonempty subset W of a vector space V over F to be a subspace of V.
13. Express the vector (1,-2,5) as a linear combination of the vectors (1,1,1), (1, 2, 3) and (2, −1, 1) in R^3.
14. If V is a vector space of finite dimension and W is a subspace of V, then prove that

    dim V/W = dim V – dim W'.
15. For any two vectors u, v in V, prove that $\|u+v\| \le \|u\| + \|v\|$.
16. Show that T: $R^2 \rightarrow R^2$ depend by T (a, b) = (a+b, a) is a vector space homomorphism.
17. Show that any square matrix can be expressed as a sum of a symmetric matrix and a skew symmetric matrix.
18. Show that the system of equations

    $x_1 + 2x_2 + x_3 = 11, \; and \; 4x_1 + 6x_2 + 5x_3 = 8 \; and \; 2x_1 + 2x_2 + 3x_3 = 19$ is inconsistent.

PART – C

Answer any TWO questions: (2 x 20 = 40 marks)

19. If $W_1$and W_2 are subspaces of a finite dimensional vector space, then prove that dim (W_1+W_2) =dim W_1 + dim W_2–dim $(W_1 \cap W_2)$.
20. If U and V are vector spaces over F and it T : U $\rightarrow$ V is a homomorphism with kernel W, then prove that U/W $\cong$ V.
21. Apply the Gram – Schmidt orthonomalization process to obtain an orthonormal basis for the subspace of R^4 generated by the vectors (1, 1, 0, 1), (1, –2, 0, 0) and (1,0, -1, 2).
22. a) Prove that the linear transformation T on V is unitary it and only it takes an orthonormal basis of V onto an orthonormal basis of V.

    b) Solve
$$x_1 + 2x_2 + x_3 + 5x_4 = 3$$
$$x_1 + 2x_2 + 2x_3 + 7x_4 = 4$$
$$x_3 + 2x_4 = 1$$
$$x_1 + x_2 + 3x_4 = 2$$

(10+10)

****