LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2016**

MT 5509 – ALGEBRAIC STRUCTURE - II

Date: 05-11-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

ANSWER ALL QUESTIONS (10 x 2 = 20)

1. If S is a subset of a vector space V and F, then show that S is a subspace of V if and only if $L(S) = S$.
2. Give an example to show that the union of two subspaces of a vector space V need not to be a subspace of V.
3. Define Nullity and Rank of a homomorphism T.
4. Define Kernel and Image of a homomorphism T.
5. State Schwarz and Triangular inequalities.
6. Define an Algebra over a field F.
7. Define skew – symmetric matrix and give an example.
8. If A and B are Hermitian, show that $AB - BA$ is skew – Hermitian.
9. If $T \in A(V)$ is a Hermitian, then prove that all its eigen values are real.
10. Define rank of a matrix.

PART – B

ANSWER ANY FIVE QUESTIONS. (5 x 8 = 40)

11. Show that a non-empty subset W of a vector space V over F is a subset of V if and only if $aw_1+bw_2 \in W$ for all $a, b \in F$ and $w_1, w_2 \in W$.
12. If S and T are subsets of a vector space V over F, then show that $L(S \cup T) = L(S) + L(T)$.
13. If V is a vector space of finite dimension and W is a subspace of V, then prove that $\dim V/W = \dim V - \dim W$.
14. Let $T: U \to V$ be a homomorphism of two vector spaces over F. Prove that the $Ker\ T$ is a subspace of U.
15. Prove that the product of two invertible linear transformations on V is itself an invertible linear transformation on V.
16. If $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 3 \\ 2 & 1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$. Calculate $(A - B)^2$.
17. If $A = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 2 & 4 \\ 5 & 6 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$. Calculate $(A+ B)^t$.
18. Prove that all the eigen values of a unitary transformation have absolute value 1.

PART – C

ANSWER ANY TWO QUESTIONS. **(2 x 20 = 40)**

19. (i) Prove that a non-empty subset W of a vector space V over F is a subspace of V if and only if W is closed under addition and scalar multiplication.

 (ii) Prove that the vector space V over F is a direct sum of two of its subspaces W_1 and W_2 if and only if $V = W_1+W_2$ and $W_1 \cap W_2 = (0)$.

20. State and prove the fundamental homomorphism theorem for vector spaces.

21. Prove that every finite – dimensional inner product space V has an orthonormal set as a basis.

22. (i) Show that a matrix A of order n over a field F is non – singular if and only if it has rank n.

 (ii) Solve the system of linear equations

$$x_1 + 2x_2 + 2x_3 = 5$$
$$x_1 - 3x_2 + 2x_3 = -5$$
$$2x_1 - x_2 + x_3 = -3$$

over the field of rational numbers.

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