LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2016**

MT 5510 – STATICS

Date: 07-11-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART –A

ANSWER **ALL** THE QUESTIONS: (10 × 2 = 20 marks)

1. Define composition and resolution of forces.
2. State the fundamental theorem of statics.
3. Define (a) like parallel forces and (b) unlike parallel forces.
4. Define (a) translation and (b) moment.
5. Write the summarization of centre of gravity.
6. Define centre of mass.
7. State Hooke's law.
8. Define unstable equilibrium and give an example.
9. Define catenary.
10. Find the shape of the catenary when the parameter is very large.

PART - B

ANSWER ANY **FIVE** QUESTIONS: (5 × 8 = 40 marks)

11. State and prove Lami's theorem.
12. Two weights P and Q are suspended from a fixed point O by strings OA and OB are kept apart by a light rod AB. If the strings OA and OB make angles α and β with the rod, show that the angle θ which the rod makes with the vertical is given by $\tan\theta = \frac{P+Q}{Q\cot\beta - P\cot\alpha}$.
13. A system of forces in the plane of ΔABC is equivalent to a single force at A, acting along the internal bisector of the angle BAC and a couple of a moment G_1. If the couple of a moments of the system about B and C are respectively G_2 and G_3, then prove that $(b+c)G_1 = bG_2 + cG_3$.
14. Find the centre of gravity of a uniform solid circular cone.
15. Find the centre of gravity of the area enclosed by the parabolas $y^2 = ax$ and $x^2 = by$ (a>0, b>0).
16. A solid homogeneous hemisphere, of radius r, has a solid right circular cone of the same material constructed on its base. The compound body rests with the curved surface of the hemisphere on a fixed sphere of radius R such that the axis of the cone is vertical. Show that the greatest height of the cone consistent with stable equilibrium is $\frac{r}{r+R}\left\{\sqrt{(3R+r)(R-r)} - 2r\right\}$.
17. Show that the length of a chain whose ends are tied together and hanging over a circular pulley of radius a so as to be in contact with two-thirds of the circumference of the pulley is $a\left\{\frac{3}{\log(2+\sqrt{3})} + \frac{4\pi}{3}\right\}$.
18. Describe the parabolic catenary.

PART - C

ANSWER ANY **TWO** QUESTIONS: (2 × 20 = 40 marks)

19. (a)Two forces of magnitudes P and $Q(P > Q)$ act on a particle and the angle between the forces is α. If the magnitudes of the forces are interchanged, show that the resultant turns through the angle $2\tan^{-1}\left(\frac{P-Q}{P+Q}\tan\frac{\alpha}{2}\right)$. (10)

(b) State and prove polygon law of forces theorem. (10)

20. (a) State and prove Varignon's theorem on moments. (10)

(b) Find the centre of gravity of a uniform solid hemisphere of radius r . (10)

21. (a) A ladder which stands on a horizontal ground leaning against a vertical wall is so loaded that its centre gravity is at the distances a and $b$from the lower and upper ends respectively. Show that if the ladder is in limiting equilibrium, its inclination θ to the horizontal is given by the $\tan\theta = \frac{a-b\mu\mu'}{(a+b)\mu}$ where μ and μ' are the coefficients of friction between the ladder and the ground and the wall respectively.

(b) A string, of length l, hangs between two points, not in the same vertical line, and the tangents at the end points are inclined at the angles α and β with the horizontal. Show that the height of one extremity above the other is $\frac{l\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}$. (10+10)

22 (a) Derive the principle of a virtual work for a system of coplanar forces acting on a rigid body.

(b) Find the work done in stretching an elastic string from its natural length l to the length l'. (15+5)

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