LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **NOVEMBER 2016**

MT 6603/MT 6600 – COMPLEX ANALYSIS

Date: 16-11-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART-A

ANSWER ALL THE QUESTIONS: (10x2=20marks)

1. Prove that the function f(z) = $\bar{z}$ is nowhere differentiable.
2. Define harmonic functions.
3. State Liouville's theorem.
4. Using Cauchy's Integral formula, evaluate $\frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3}dz$ where C is |z|=4.
5. Find the poles of $f(z)=\frac{z^2-2z+3}{z-2}$
6. State Maximum Modulus theorem.
7. Find the residue of $\frac{ze^z}{(z-1)^3}$ at its poles.
8. What are the different types of singularities ?
9. Define conformal mapping
10. Define a bilinear transformation.

PART-B

ANSWER ANY FIVE QUESTIONS: (5x8=40marks)

11. Prove that $f(z)=\sin x\cosh y+i\cos x\sinh y$ is differentiable at every point.
12. Evaluate $\int_C \frac{e^z}{(z+2)(z+1)^2}dz$ where C is $|z|=3$.
13. State and prove fundamental theorem of algebra.
14. Let $f(z)$ be a function having a as an isolated singularpoint. Then prove that the following are equivalent.
    i) a is a pole of order r for $f(z)$.
    ii) $f(z)$ can be written in the form $f(z)=\frac{1}{(z-a)^r}\theta(z)$ where $\theta(z)$ has a removable singularity at $z=a$ and $\lim_{z\to a}\theta(z)\neq 0$.
    iii) a is a zero of order r for $\frac{1}{f(z)}$.
15. State and prove Rouche's theorem.

16. Evaluate by using Cauchy's integral formula $\int_C \frac{z+1}{z^2+2z+4}dz$ where C is the circle $|z+1+i| = 2$

17. Prove that any bilinear transformation can be expressed as a product of translation, rotation, magnification or contraction and inversion.

18. Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ onto $w_1 = 1, w_2 = i, w_3 = -1$ respectively.

PART-C

ANSWER ANY TWO QUESTIONS: (2 x 20=40marks)

19. a) Derive C.R equations in polar coordinates.

b) If $f(z)$ is analytic prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (12+8)

20. a) State and prove Cauchy's integral theorem.

b) Evaluate $\int_C \left(\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\right)$ where C is the circle $|z| = 3$. (12+8)

21. a) State and prove Laurent's theorem.

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$. (12+8)

22. a) Using the method of contour integration evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}dx$

b) Find the bilinear transformation which maps $-1, 0, 1$ of the z-plane onto $-1, -i, 1$ of the w-plane. Show that under this transformation the upper half of the w-plane maps onto the interior of the unit circle. (12+8)