

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600 034

B.Sc. DEGREE EXAMINATION - MATHEMATICS SIXTH SEMESTER - NOVEMBER 2016 MT 6608 - DISCRETE MATHEMATICS

Date: 16-11-2016 Dept. No. Max.: 100 Marks

Time: 09:00-12:00

PART – A

ANSWER ALL QUESTIONS

 $(10 \times 2 = 20)$

- 1. Give the truth table for the disjunction operation.
- 2. Write in symbolic form: The crop will be destroyed if there is a flood.
- 3. What is a disjunctive normal form?
- 4. Define Tautology.
- 5. What is a monoid?
- 6. Define semigroup homomorphism.
- 7. Define a lattice.
- 8. Give the Idempotent and Absorption laws of Lattices.
- 9. Define a Boolean Algebra.
- 10. What is a sub Boolean algebra?

PART - B

ANSWER ANY FIVE QUESTIONS.

 $(5 \times 8 = 40)$

- 11. Construct the truth table for the formula $(P \land (P \rightarrow Q)) \rightarrow Q$.
- 12. Show that $((P \lor Q) \land \neg (\neg (P \land (\neg Q \lor \neg R))) \lor (\neg P \land \neg Q) \lor (\neg P \land \neg R)$ is a tautology.
- 13. Find the principal conjunctive normal form of $(P \land Q) \lor (\neg P \land R)$.
- 14. Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$.
- 15. Prove that the composition of semigroup homomorphisms is also a semigroup homomorphism.
- 16. Prove the isotonicity properties of lattices.
- 17. Prove the distributive inequalities on Lattices.
- 18. Define the switching algebra and show that it is a Boolean algebra.

PART - C

ANSWER ANY TWO QUESTIONS.

 $(2 \times 20 = 40)$

- 19. (i) Show that $(\neg P \land (\neg Q \land R)) \lor (Q \land R) \lor (P \land R) \Leftrightarrow R$.
 - (ii) Show that if any two formulas are equivalent then their duals are also equivalent to each other.
- 20. (i) Show that the following premises are inconsistent:
 - I. If Jack misses many classes through illness, then he fails in high school.
 - II. If Jack fails high school, then he is uneducated.
 - III. If Jack reads a lot of books, then he is not uneducated.
 - IV. Jack misses many classes through illness and reads a lot of books.
 - (ii) Establish the existence of a homomorphism from the free semigroup with n generators to any semigroup with n generators.
- 21. (i) Prove that the set of idempotent elements of any commutative monoid M form a submonoid.
 - (ii) Define a sublattice of a lattice and show that every interval of a lattice is a sublattice.
- 22. (i) Write the following Boolean expressions in an equivalent sum of- products canonical form in three variables x_1 , x_2 and x_3 : (a) $x_1 * x_2$; (b) $x_1 \oplus x_2$; (c) $(x_1 \oplus x_2)' * x_3$.
 - (ii) What is binary valuation process? How do you use this process to show the equivalence of two Boolean expressions?
