LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2017**

17/16UMT1MC02– ANALYTICAL GEOMETRY OF 2D, TRIG. MATRICES

Date: 08-11-2017 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A
Answer ALL questions(10 X 2 = 20)

1. Write the expansion for $\sin n\theta$.

2. If $x = \cos\theta + i\sin\theta$ find $x^n + \frac{1}{x^n}$.

3. Show that $\sin(ix) = i\sinh x$ and $\cos(ix) = \cosh x$.

4. Find the value of Log(4 + 3i).

5. State Cayley Hamilton theorem.

6. Find the eigen values of the matrix $\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$.

7. Find the pole of the line Ax + By + C = 0 with respect to the parabola $y^2 = 4ax$.

8. Define the conjugate diameters of the ellipse.

9. Write the equation to theasymptotes of thehyperbola$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

10. Find the eccentricity of the conic $\frac{12}{r} = 4 + \sqrt{3}\cos\theta + 3\sin\theta$.

PART – B
Answer any FIVE questions(5 X8 = 40)

11. Expand $\cos 6\theta$ in terms of $\sin\theta$.

12.If $\sin\theta = 0.5033$, show that θ is approximately $30° 13' 6''$.

13. Separate into real and imaginary parts $\tan^{-1}(x+iy)$

14. Prove that$sinh^{-1}x = log_e(x + \sqrt{x^2+1})$.

15. Show that the matrix $\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ satisfies Cayley Hamilton theorem.

16. Find the characteristic roots of the matrix $= \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

17. Prove that in an ellipse, the tangents at the extremities of a diameter are parallel to the chords bisected by the diameter.

18. Find the asymptotes of the hyperbola $3x^2 – 5xy – 2y^2 + 17x + y + 14 = 0$.

PART– C

Answer Any TWO Questions(2 X 20 = 40)

19. a. If $\tan(x+iy) = u+iv$, prove that $\frac{u}{v} = \frac{\sin 2x}{\sinh 2y}$.

b. Expand $\sin^3\theta\cos^4\theta$ in terms of sines of multiples of θ. **(8 + 12)**

20. a. If $\tan(\alpha+i\beta) = x+iy$ prove that $x^2+y^2+2x\cot 2\alpha = 1$

b. If $\log\sin(\theta+i\phi) = A+iB$, Show that

(i) $2e^{2A} = \cosh 2\phi - \cos 2\theta$ (ii) $\tanh\phi = \tan\theta\tan B$ **(8 + 12)**

21. Diagonalize the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

22. a. A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ whose centre is C meets the circle

$x^2+y^2 = a^2+b^2$ at Q and Q' . Prove that CQ and CQ' are conjugate diameters of the ellipse.

b. Find the locus of the foot of the perpendiculars drawn from the pole to the tangents

to the circle $r = 2a\cos\theta$ **(10 + 10)**

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