



**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**

**B.Sc. DEGREE EXAMINATION – MATHEMATICS**

**FIRST SEMESTER – NOVEMBER 2017**

**MT 1500 – ALGEBRA, ANALY. GEO., CALCULUS & TRIGONOMETRY**

Date: 14-11-2017

Dept. No.

Max. : 100 Marks

Time: 01:00-04:00

**PART-A**

**Answer all the questions**

**(10 x 2=20)**

1. Find the  $n^{\text{th}}$  derivative of  $y = \log(ax + b)$
2. Find the sub tangent and subnormal of the parabola  $y^2 = 4ax$ .
3. What is the radius of curvature of the curve  $x^4 + y^4 = 2$  at the point (1,1)?
4. Define involute.
5. Find the equation, with rational coefficients one of whose roots is  $\sqrt{5} + \sqrt{2}$ .
6. Calculate the sum of the cubes of the roots of the equation  $x^4 + 2x + 3 = 0$
7. Show that  $\cosh^2 x - \sinh^2 x = 1$ .
8. Write down the expansion for  $\cos n_{\theta}$ .
9. Write the polar equation of a conic.
10. Find the asymptotes of the hyperbola  $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$ .

**PART - B**

**Answer any FIVE questions:**

**(5 X 8 = 40)**

11. Find the  $n^{\text{th}}$  differential coefficient of  $\cos x \cdot \cos 2x \cdot \cos 3x$ .
12. Find the slope of the tangent with the initial line for the cardioid  $r = a(1 - \cos \theta)$  at  $\theta = \frac{f}{6}$ .
13. Prove that the radius of curvature at any point of the cycloid  $x = a(\theta + \sin \theta)$  and  $y = a(1 - \cos \theta)$  is  $4a \cos \frac{\theta}{2}$ .
14. Solve the equation  $27x^3 + 42x^2 - 28x - 8 = 0$  whose roots are in G.P.
15. Express  $\frac{\sin 6\theta}{\sin \theta}$  in terms of  $\cos \theta$ .
16. Separate into real and imaginary parts  $\tan^{-1}(x + iy)$ .
17. If  $P$  and  $Q$  are extremities of two conjugate diameters of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and  $S$  is a focus, then prove that  $PQ^2 - (SP - SQ)^2 = 2b^2$ .
18. Show that in a conic the semi-latus rectum is the harmonic mean between the segments of a focal chord.

**PART - C**

**Answer any TWO questions:**

**(2 x 20=40)**

19. If  $y = \sin(m \sin^{-1} x)$ , prove that  $(1 - x^2)y_2 - xy_1 + m^2y = 0$  and

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$$

20. a) For the curves  $x^2 = 4y$  and  $y^2 = 4x$ , find the angle of intersection.

b) Find the evolute of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . **(10+10)**

21. Solve the equation  $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$ .

22. a) Sum to infinity the series  $\cos r + \frac{1}{2} \cos(r + s) + \frac{1}{2} \cdot \frac{3}{4} \cos(r + 2s) + \dots$

b) If  $e$  and  $e_1$  are two extremities of hyperbola and its conjugate show that  $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$  **(10+10)**

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