LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FIRSTSEMESTER – NOVEMBER 2017

MT 1816- REAL ANALYSIS

Date: 04-11-2017 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all the questions.

1. a)Prove that$\int_{a_-}^{b} f d\alpha \leq \int_{a}^{b^-} f d\alpha$.

OR

(b) State and prove the fundamental theorem of calculus. (5)

(c) State and prove a necessary and sufficient condition for a bounded real valued function to be a Riemann-Steiltjes integrable. (15)

OR

(d) (i) Any monotone function f: [0, 1]→R is Riemann Integrable. Justify.

(ii) Let $f \in \Re(\alpha)$ on [a, b]$m \leq f \leq M$, φ be continuous on [m,M] and $h(x) = \varphi(f(x))$ on [a,b]. Then prove that $h \in \Re(\alpha)$ on [a, b]. (6+9)

2. (a) State and prove the Cauchy criterion for uniform convergence of sequence of functions.

(OR)

(b) Prove that for$f_n(x) = n^2x(1-x^2)^n, 0 \leq x \leq 1, n = 1,2 \ldots$,

$\int_0^1 (\lim_{n\to\infty} f_n(x))\, dx \neq \lim_{n\to\infty} \int_0^1 f_n(x) dx$. (5)

(c) State and prove the Stone-Weierstrass theorem.

(OR)

(d) If $\{f_n\}$ is a sequence of differentiable functions on [a, b] such that $\{f_n(x_0)\}$ converges for $x_0 \in [a, b]$ and $\{f_n'\}$ converges uniformly on [a, b] then prove that $\{f_n\}$ converges uniformly on [a, b] to a function f and $\lim_{n\to\infty} f_n'(x) = f'(x)$ (15)

3. (a) Let $S = \{\varphi_0, \varphi_1, \varphi_2, \ldots\}$, where $\varphi_0(x) = \frac{1}{\sqrt{2\pi}}$, $\varphi_{2n-1}(x) = \frac{\cos nx}{\sqrt{\pi}}$ and $\varphi_{2n}(x) = \frac{\sin nx}{\sqrt{\pi}}$, for n = 1, 2…. Prove that S is orthnormal on any interval of length 2π.

OR

(b) State and prove the Bessel's Inequality. (5)

(c) (i) State and prove Riesz-Fischer theorem.

(ii) State and prove Riemann-Lebesgue lemma. (8+7)

OR

(d) (i) Assume that $f \in L[0,2\pi]$ and suppose that f is periodic with period2π. Let $\{s_n\}$ denote the sequence of partial sums of the Fourier series generated by f, $s_n(x) = \frac{a_0}{2} + \sum_{k=0}^{\infty}(a_k coskx + (b_k sinkx)$, n=1,2,…. Then prove that $s_n(x) = \frac{2}{\pi}\int_0^{\pi} \frac{f(x+t)+f(x-t)}{2} D_n(t) dt$ where D_n is called Dirichlet's Kernel.

(ii) If $g(0+)$ exists and for some δ>0, the Lebesgue integral $\int_0^b \frac{g(t)-g(0+)}{t} dt$ exists, then prove that $\lim_{\alpha\to\infty} \frac{2}{\pi}\int_0^b g(t) \frac{sin\alpha t}{t} dt = g(0+)$. (8+7)

4. (a)Prove that L(R^n,R^m) is a metric space where the distance between A and B is defined as $\|A - B\|$.and A,B ∈L(R^n,R^m) (5)

OR

(b) State and prove the fixed point theorem. (5)

(c) Suppose f is a ζ' mapping of an open set $E \subset R^n$ into R^n and f'(a) is invertible for some $a \in E$ and f(a)=b. Then prove that there exists open sets U and V in R^n such that $a \in U$, $b \in V$, f is one-to-one on U and f(U)=V. Also prove that if g is the inverse of f then $g \in \zeta'(V)$.

OR

(d) State and prove the implicit function theorem. (15)

5. (a) Define heat flow and the heat equation.

OR

(b) Explain rectilinear coordinate system with algebraic and geometric approach. (5)

(c) Derive the expression for Newton's Law of Cooling.

(OR)

(d) Derive the D' Alembert's wave equation for a vibrating string. (15)

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