LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2017**

MT 3501 – ALGEBRA, CALCULUS AND VECTOR ANALYSIS

Date: 04-11-2017 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer all questions: **(10 X 2 = 20)**

1. Evaluate $\int_0^a \int_0^b \left(x^2 + y^2\right) dxdy$.
2. Evaluate $\int_0^1 x^7 (1-x)^8 \, dx$.
3. Obtain a PDE by eliminating a and b from $z = (x+a)(y+b)$.
4. Solve $pq = 1$.
5. Find ∇¤ , if ¤ $= xy^2 + yz^3$.
6. Find the unit vector normal to the surface $x^2 + 2y^2 + z^2 = 7$ at $(1,-1,2)$.
7. Find $L\left(t^3 - 3t^2 + 2\right)$.
8. Find $L\left(t \sin t\right)$.
9. Show that $n(n+1)(2n+1)$ is divisible by 6.
10. Find the remainder when 2^{1000} is divisible by 17.

PART – B

Answer any five questions: **(5 X 8 = 40)**

11. Evaluate $\int\int xy\,dxdy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$.
12. Evaluate $\int_0^\infty e^{-x^2}$ using Gamma function.
13. Eliminate the arbitrary function f and ¤ from the relation $z = f(x+ay)+$¤$(x-ay)$.
14. Solve $p \tan x + q \tan y = \tan z$.
15. Compute the divergence and curl of the vector $\vec{F} = xyz\,\hat{i} + 3x^2y\,\hat{j} + \left(xz^2 - y^2z\right)\hat{k}$ at $(1,2,-1)$.
16. Prove that (i) $L\left(e^{-at}\right) = \frac{1}{s+a}$, provided $s + a > 0$

    (ii) $L(\cos at) = \frac{s}{s^2 + a^2}$

17.Find $L^{-1}\left(\frac{1}{(s+1)(s^2+2s+2)}\right)$.

18.Find the remainder obtained in dividing 2^{46} by 47.

PART – C

Answer any two questions: **(2 X 20 = 40)**

19.(i) Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xydxdy$ and evaluate it.

(ii) Prove that $(m,n)=\frac{\Gamma m\,\Gamma(n)}{\Gamma(m+n)}$. **(10+10)**

20.(i) Find the general solution of $\left(y^2+z^2\right)p-xyq=-xz$

(ii) Obtain a complete integral of $xp^2-ypq+y^3q-y^3z=0$. **(10+10)**

21.(i) Verify Gauss- Divergence theorem for the function $\vec{F}=2xz\,\hat{i}+yz\,\hat{j}+z^2\,\hat{k}$ over the upper half of the sphere $x^2+y^2+z^2=a^2$.

(ii) Find the smallest number with 18 divisors. **(15+5)**

22.(i) Solve the equation $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=4$ subject to $y=2,\frac{dy}{dx}=3$ when $x=0$.

(ii) State and prove Wilson's theorem. **(10+10)**

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